A Technology-Gap Model of 'Premature' Deindustrialization

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Introduction

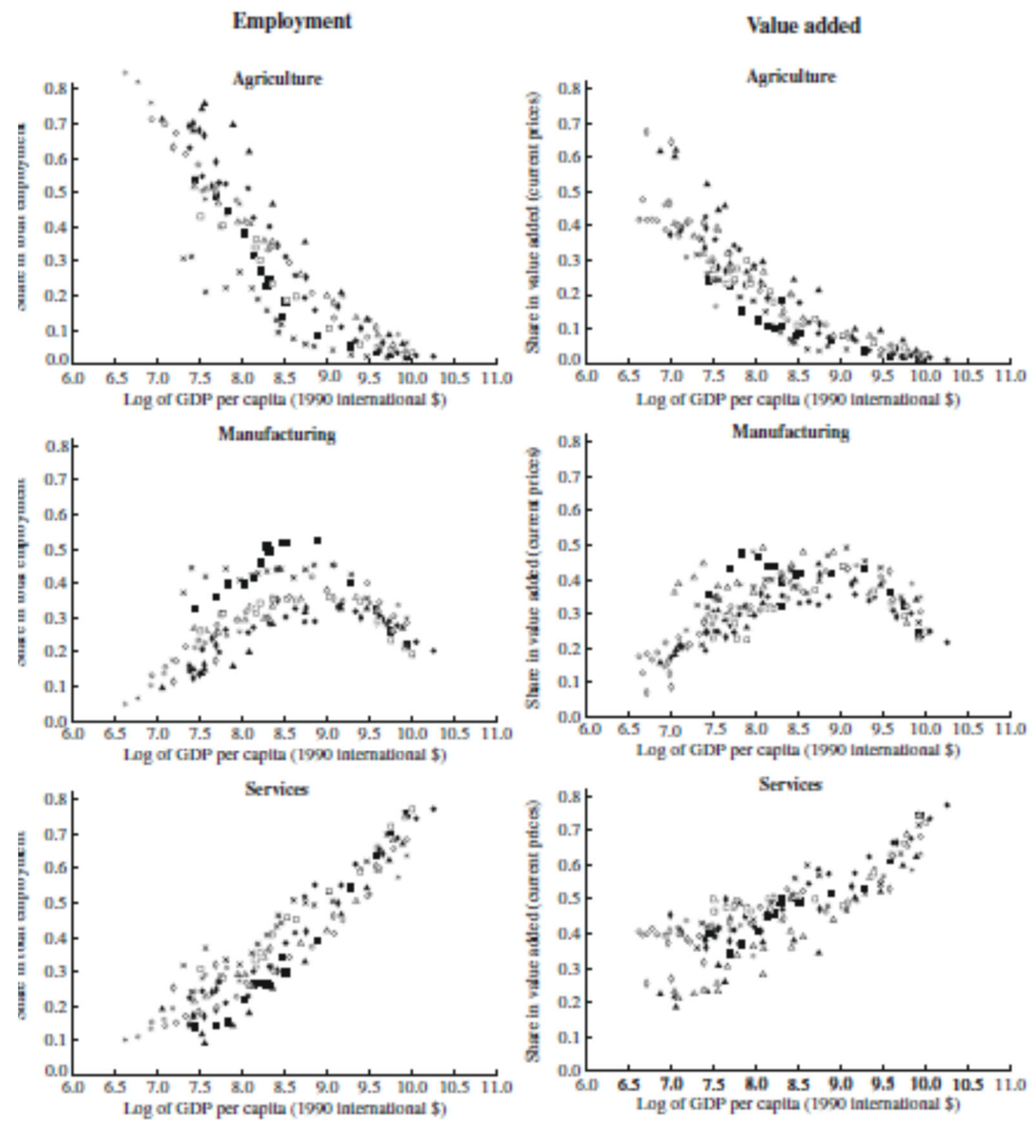
Structural Change

As per capita income rises, employment or value-added shares

- Fall in Agriculture
- Rise in Services
- Rise and Fall in Manufacturing

From Herrendorf-Rogerson-Valentinyi (2014)

Evidence from Long Time Series for the Currently Rich Countries (Belgium, Finland, France, Japan, Korea, Netherlands, Spain, Sweden, United Kingdom, and United States) 1800-2000



Premature Deindustrialization: Rodrik (JEG 2016)

Late industrializers reach their M-peak and start deindustrializing

- *Later* in time
- *Earlier* in per capita income
- with the *lower* peak M-sector shares,

compared to early industrializers.

Rodrik (2016) focuses on documenting the patterns, without offering a causal explanation or making normative statements. But

- He speculates that globalization may be a cause.
- The word, "premature" seems to suggest some types of inefficiency that might call for government interventions.

In our model, "premature" deindustrialization occurs in the efficient equilibrium of a closed economy.

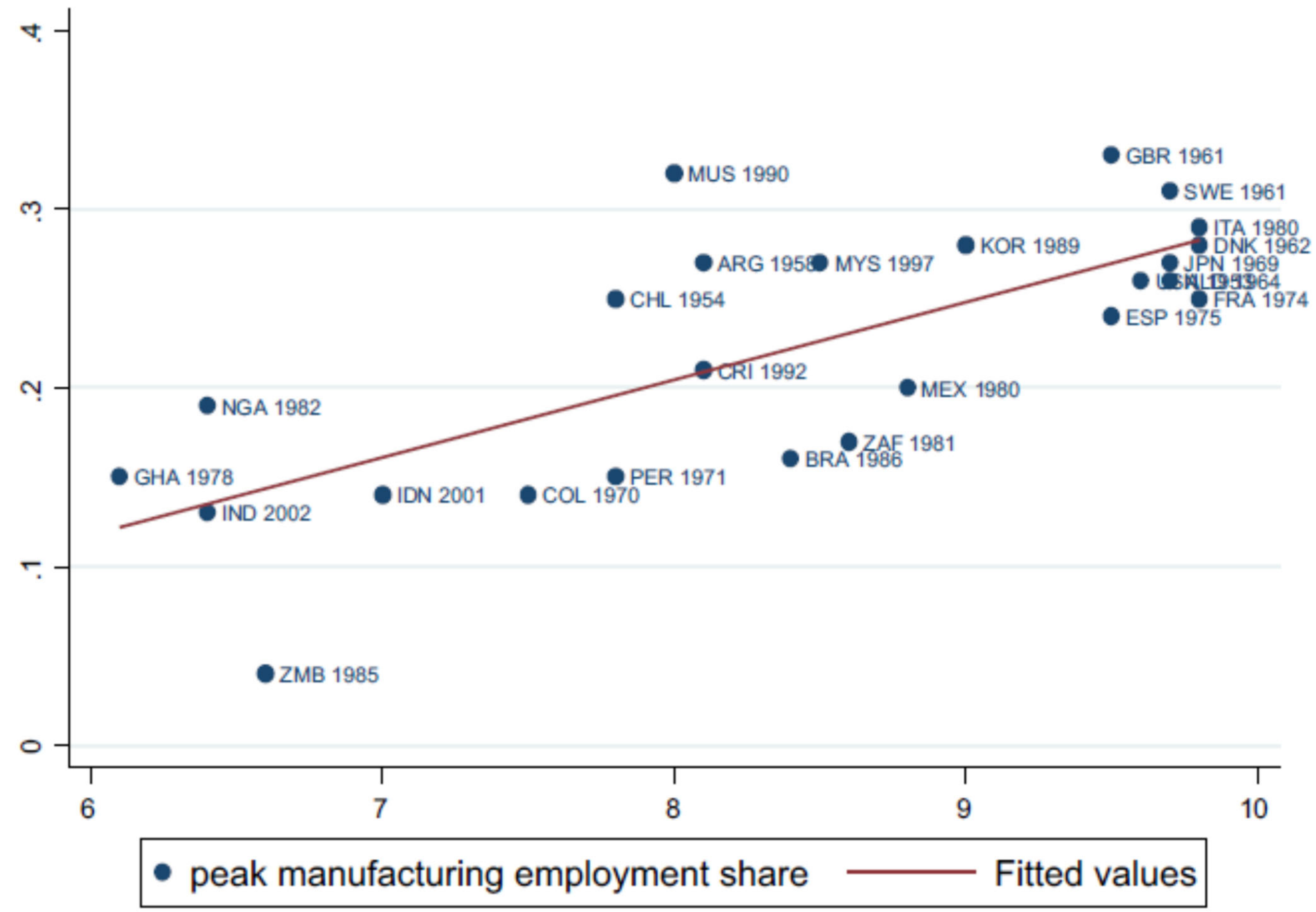


Fig. 5 Income at which manufacturing employment peaks (logs)

This Paper: A Parsimonious Model of Premature Deindustrialization (PD)

3 Goods/Sectors: 1=(**A**)griculture, 2=(**M**)anufacturing, 3=(**S**)ervices, *homothetic CES with gross complements* ($\sigma < 1$)

Frontier Technology: $\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$, with $g_1 > g_2 > g_3 > 0$ ⇒ a decline of A, a rise of S, and a hump-shaped of M in each country through **the Baumol (relative price) effect**, as in Ngai-Pissarides (2007)

Actual Technology Used: $A_j(t) = \bar{A}_j(t - \lambda_j)$ due to **Adoption Lags,** $(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$

- $\lambda \geq 0$, **Technology Gap,** country-specific, as in Krugman (1985); their ability to adopt the frontier technologies.
- $\theta_j > 0$: sector-specific, unlike Krugman (1985); how much λ affects the adoption lag and productivity in each sector.

$$A_j = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t} = \bar{A}_j(0)e^{-g_j\theta_j\lambda}e^{g_j t} \Longrightarrow \frac{\partial}{\partial\lambda}\ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

λ has **no "growth" effect, but negative "level" effects,** *proportional* to $\theta_j g_j$ in sector-j

Key Mechanisms

- θ_j magnifies the impact of the technology gap on the adoption lag: $\frac{\partial}{\partial\theta_j}\left(\frac{\partial\lambda_j}{\partial\lambda}\right) > 0$ (*supermodularity*)
- g_j magnifies the (negative) impact of the adoption lag on productivity: $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial\lambda_j}\ln e^{-\lambda_j g_j}\right) < 0$ *(log-submodularity)*

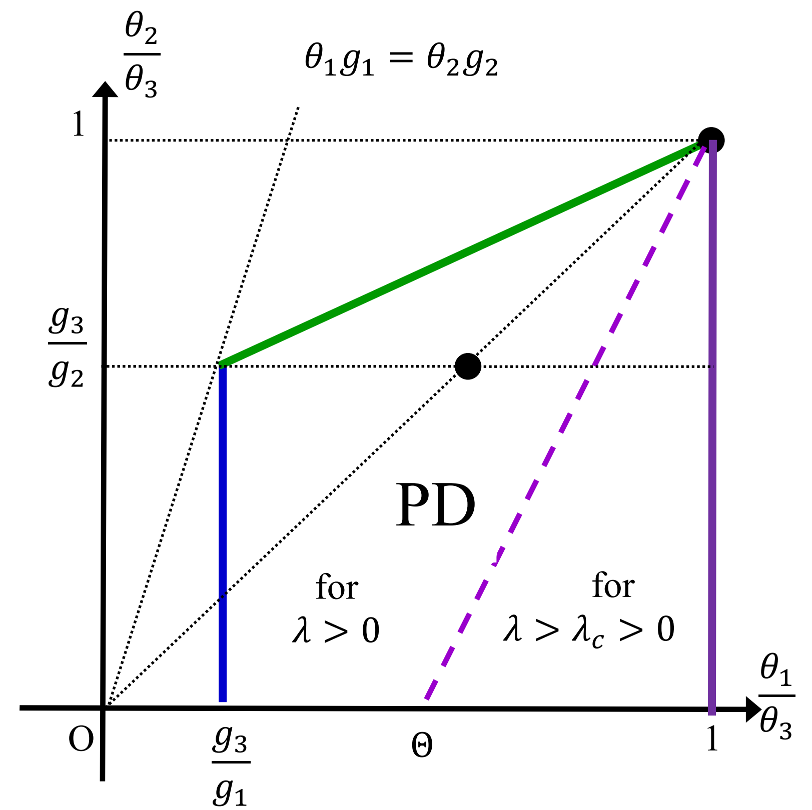
Main Results: Conditions for PD, defined as "A high-λ country reaches its peak later in time, with lower peak M-share at lower peak time per capita income."

i) $\theta_1 g_1 > \theta_3 g_3$: cross-country productivity difference larger in A than in S.
High relative price of A/low relative price of S in a high-λ country causes a delay.

ii) $\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}$: technology adoption takes not too long in M.
Not too high relative price of M in a high-λ country keeps the M-share low.

Under the above conditions,

iii) $\theta_1 < \theta_3$: Technology adoption takes longer in S than in A.
Longer adoption lag in S in a high-λ country causes "premature" deindustrialization.



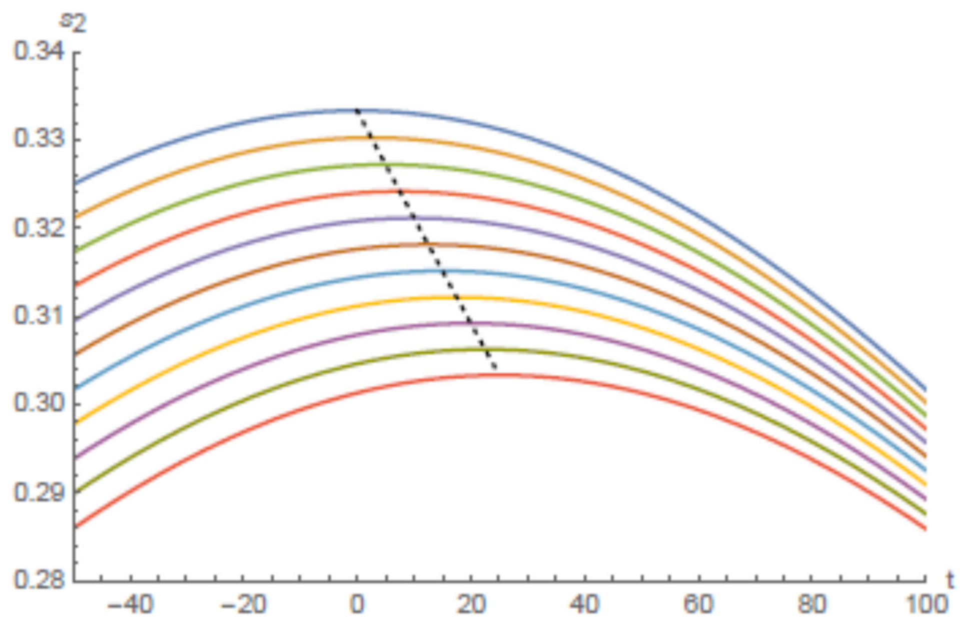
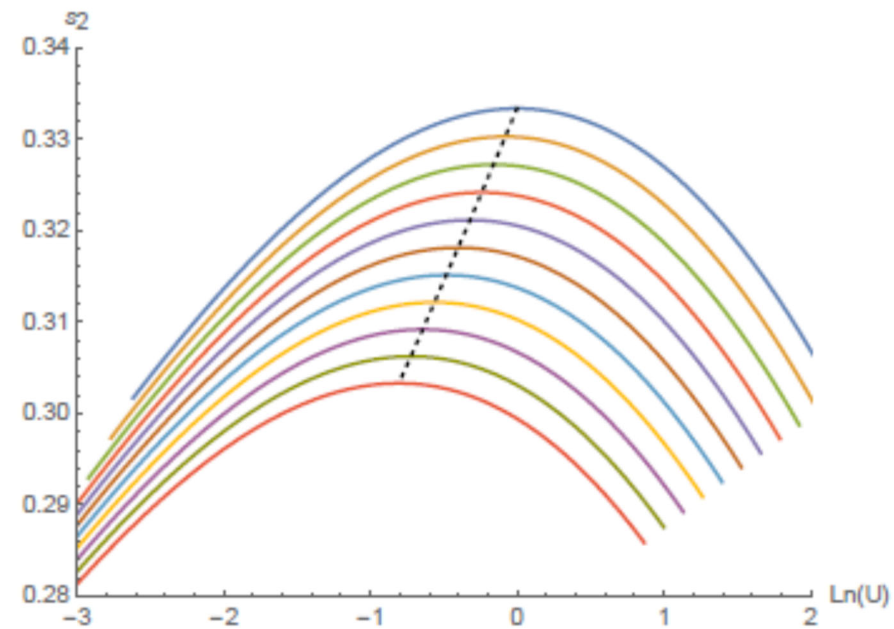
Some Implications:
No PD if $\theta_1 = \theta_2 = \theta_3$. Latecomers would follow the same path with a delay.

i) & ii) $\Rightarrow \theta_1 g_1 > \theta_2 g_2, \theta_3 g_3$: Cross-country productivity difference the largest in A.
The sign of $\theta_2 g_2 - \theta_3 g_3$ can be positive or negative; slightly negative to match the finding of Rodrik (2016; Table 10)

ii) & iii) $\Rightarrow \theta_1, \theta_2 < \theta_3$: Technology adoption takes longest in S.

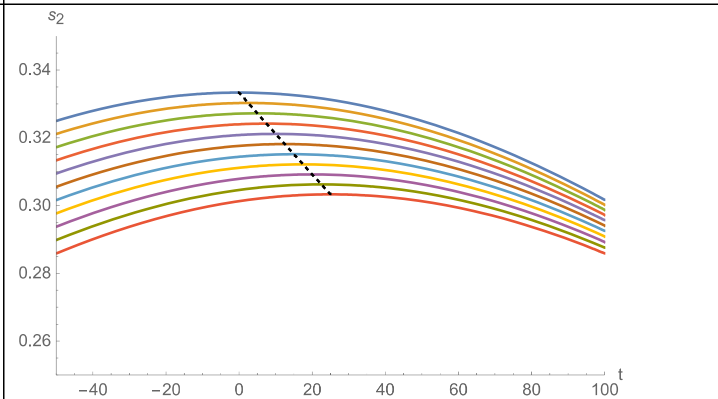
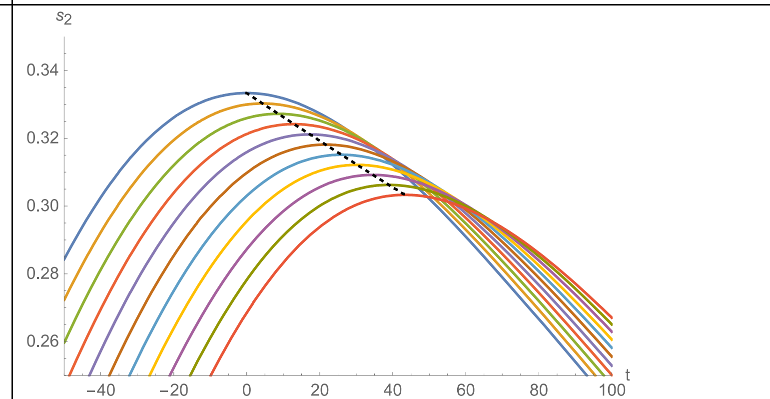
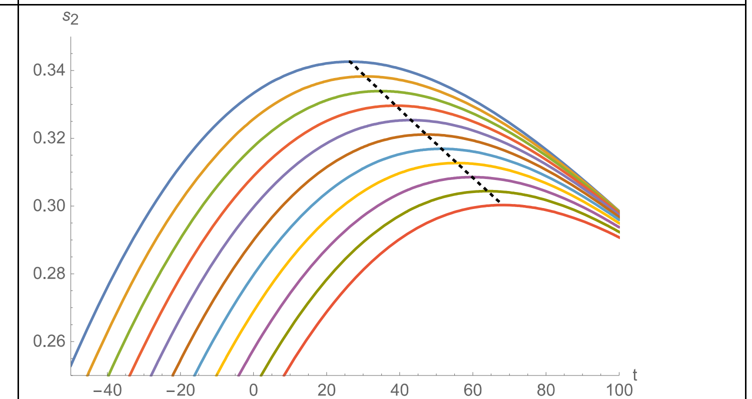
A Numerical Illustration.

$\theta_1 = \theta_2 < \theta_3 = 1$ with $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\sigma = 0.6$; Labor share $= 2/3$. We set the other parameters, w.l.o.g., so that the peak time, $\hat{t} = 0$ and the peak time income per capita, $U(\hat{t}) = 1$ if $\lambda = 0$.

Example 2a	$\left(t, s_2(t)\right)$	$\left(\ln U(t), s_2(t)\right)$
$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$ $\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$		
λ=0 λ=10 λ=20 λ=30 λ=40 λ=50 λ=60 λ=70 λ=80 λ=90 λ=100		

1st Extension: Adding **the Engel Effect** with **Nonhomothetic CES** (Comin-Lashkari-Mestieri)

Nonhomotheticity changes the shape of trajectories greatly, but not on how technology gaps, λ, affects the peak values.

<table>
<tr><td></td><td>Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)</td><td>Unbiased case($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$)</td><td>Biased case($\varepsilon_1 = .4 < \varepsilon_2 = 1.2 < \varepsilon_3 = 1.4$)</td></tr>
<tr><td>$(t, s_2(t))$</td><td>
</td><td>
</td><td>
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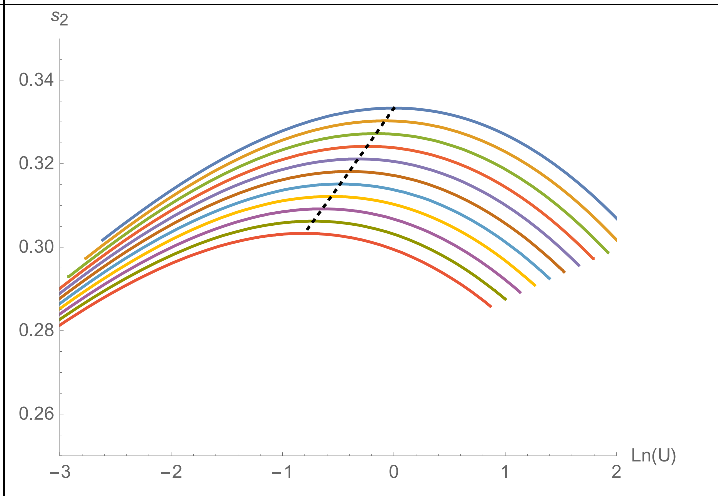
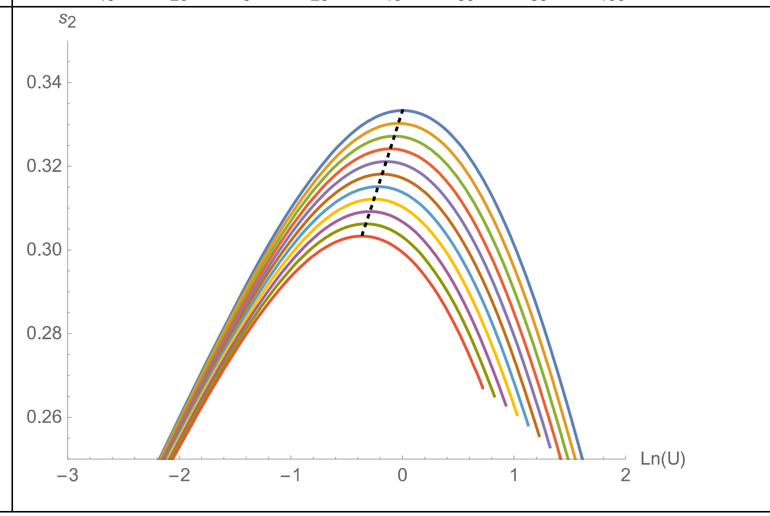
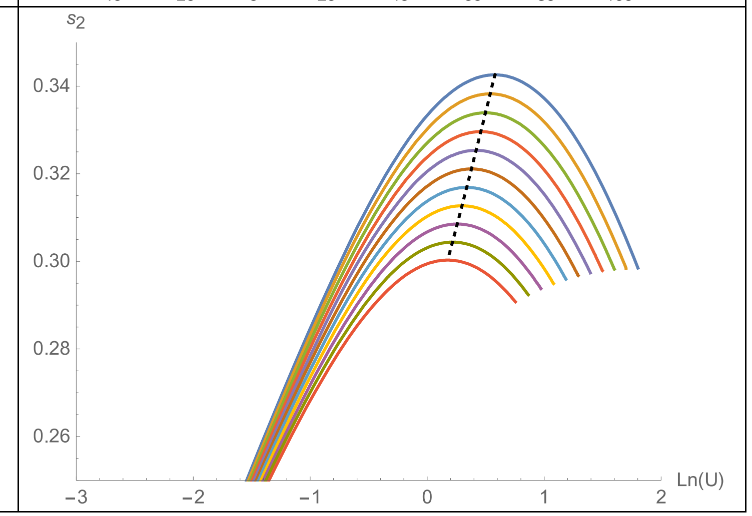
<tr><td>$(\ln U(t), s_2(t))$</td><td>
</td><td>
</td><td>
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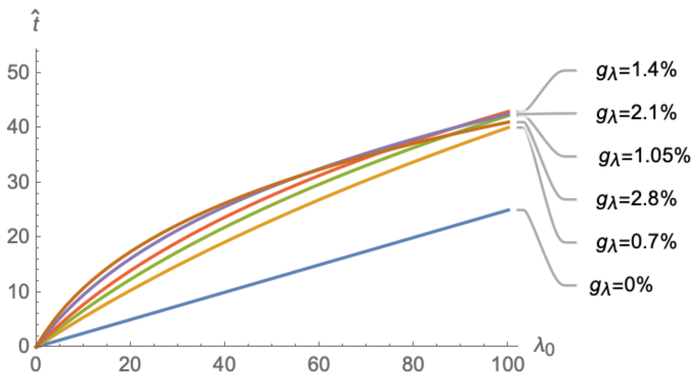
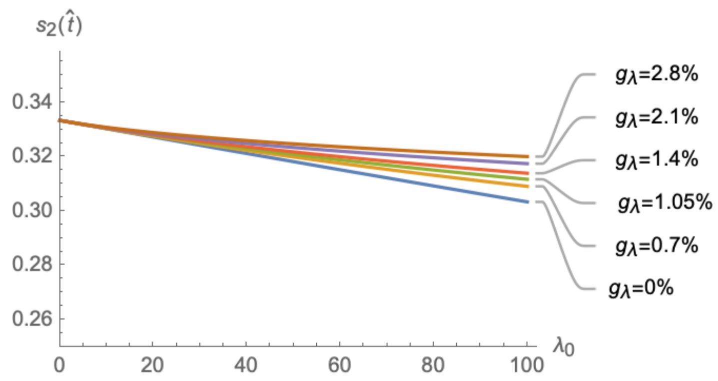
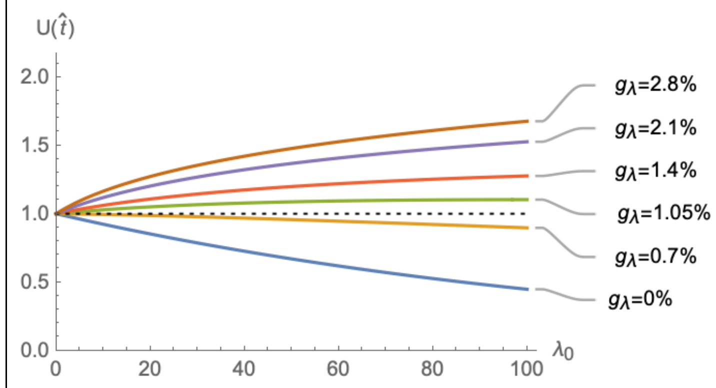
We also show that the Engel effect *alone* could not generate PD *without counterfactual implications*.

2nd Extension: Introducing Catching-up

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \qquad \text{where} \;\; \lambda_t = \lambda_0 e^{-g_\lambda t},$$

Countries differ only in the *initial* value, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,** $g_\lambda > 0$

Peak Time	Peak M-Share	Peak time Per Capita Income
		

Higher-λ countries

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large.

(**Very Selective) Literature Review.** Herrendorf-Rogerson-Valentinyi (14) for a survey on structural change.

Related to The Baseline Model

Premature Deindustrialization, Dasgupta-Singh (06), Palma (14), **Rodrik (16)**
The Baumol Effect: Baumol (67), **Ngai-Pissarides (07),** Nordhaus (08)
Cross-country heterogeneity in technology development

- *Distance to the frontier:* **Krugman** (85), Acemolgu-Aghion-Zilibotti (06)
- *Log-supermodularity:* **Krugman** (85), Matsuyama (05), Costinot (09), Costinot-Vogel (15)
- *Productivity difference across countries the largest in A:* Caselli (05), Gollin et.al. (14, AERP&P)
- *Small adoption lags in M;* Rodrik (2013)

Related to Two Extensions

The Engel Effect (Nonhomotheticity); Murphy et.al. (89), Matsuyama (92,02), Kongsamut et.al. (01), Foellmi-Zweimueller (08), Buera-Kaboski (09,12), Boppart (14), **Comin-Lashkari-Mestieri (21),** Matsuyama (19), Lewis et.al. (21), Bohr-Mestieri-Yavuz (21)
Catching-Up/Technology Diffusion: Acemoglu (08), Comin-Mestieri (18)

The Issues We Abstract From

Sector-level productivity growth rate differences across countries: Huneeus-Rogerson (20)
Open economy implications: Matsuyama (92,09), Uy-Yi-Zhang (13), Sposi-Yi-Zhang (19), Fujiwara-Matsuyama (Work in Progress)
Endogenous growth, externalities, Matsuyama (92),
Sectoral wedges/misallocation: Caselli (05), Gollin et.al. (14 QJE) and many others
Nominal vs. Real expenditure; Employment vs. Value Added shares; Compatibility with aggregate balance growth, investment vs consumption, sector-specific factor intensities, skill premium, home production, productivity slowdown, etc.

Structural Change, the Baumol Effect, and Adoption Lags

Three Complementary Goods/Competitive Sectors, $j = 1, 2, 3$

Sector-1 = **(A)**griculture, Sector-2 = **(M)**anufacturing, Sector-3 = **(S)**ervices.

Demand System: L Identical HH, each supplies 1 unit of mobile labor at w; κ_j units of factor specific to j at ρ_j.

Budget Constraint:

$$\sum_{j=1}^{3} p_j c_j \le E \equiv w + \sum_{j=1}^{3} \rho_j \kappa_j = \frac{1}{L} \sum_{j=1}^{3} p_j Y_j$$

CES Preferences:

$$U(c_1, c_2, c_3) = \left[\sum_{j=1}^{3} (\beta_j)^{\frac{1}{\sigma}} (c_j)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

with $\beta_j > 0$ and $0 < \sigma < 1$ (gross complementarity)

Expenditure Shares:

$$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j (p_j)^{1-\sigma}}{\sum_{k=1}^{3} \beta_k (p_k)^{1-\sigma}} = \beta_j \left(\frac{E/p_j}{U}\right)^{\sigma-1}$$

Three Competitive Sectors: Production

Cobb-Douglas

$$Y_j = \tilde{A}_j\left(\kappa_j L\right)^{\alpha}\left(L_j\right)^{1-\alpha}$$

$\tilde{A}_j > 0$: the TFP of sector-j; $\alpha \in [0,1)$ the share of specific factor.

Employment Share

$$s_j \equiv \frac{L_j}{L}; \qquad \sum_{j=1}^{3} s_j = 1$$

Output per worker
Output per capita

$$\frac{Y_j}{L_j} = A_j\left(s_j\right)^{-\alpha}; \qquad \frac{Y_j}{L} = A_j\left(s_j\right)^{1-\alpha}$$

where $A_j \equiv \tilde{A}_j\left(\kappa_j\right)^{\alpha}$.

With Cobb-Douglas, $wL_j = (1-\alpha)p_j Y_j$, implying the employment shares equal to

Value-Added Shares

$$\frac{p_j Y_j}{EL} = \frac{p_j Y_j}{\sum_{k=1}^{3} p_k Y_k} = s_j = \frac{L_j}{L}$$

Equilibrium: The expenditure shares are equal to the employment and value-added shares.

$$m_j = \frac{p_j Y_j}{EL} = s_j$$

which lead to

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j\right]^{-a}}{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}}$$

Per Capita Income

$$U = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}} A_j\right]^{-a}\right\}^{-\frac{1}{a}}$$

where

$$a \equiv \frac{1-\sigma}{1-\alpha(1-\sigma)} = -\frac{\partial \log\left(s_j/s_k\right)}{\partial \log\left(A_j/A_k\right)} > 0,$$

which captures how much relatively high productivity in a sector contributes to its relatively low equilibrium share. α magnifies this effect by increasing a.

Productivity Growth:

$$A_j(t) = \bar{A}_j(t-\lambda_j) = \bar{A}_j(0)e^{g_j(t-\lambda_j)} = \bar{A}_j(0)e^{-\lambda_j g_j}e^{g_j t}$$

$\bar{A}_j(t) = \bar{A}_j(0)e^{g_j t}$: **Frontier Technology** in j, with a constant **growth rate** $g_j > 0$.

$A_j(t) = \bar{A}_j(t-\lambda_j)$; λ_j = **Adoption Lag** in j.

- g_j and λ_j are sector-specific.
- λ_j has **no "growth" effect.**
- λ_j has **the "level" effect,** $e^{-\lambda_j g_j}$, which is decreasing in λ_j and the effect is proportional to g_j

Key: Log-submodularity, $\frac{\partial}{\partial g_j}\left(\frac{\partial}{\partial \lambda_j}\ln e^{-\lambda_j g_j}\right) < 0$: g_j magnifies the negative effect of the adoption lag on productivity

A large adoption lag would not matter much in a sector with slow productivity growth.
Even a small adoption lag would matter a lot in a sector with fast productivity growth.

$$U(t) = \left\{\sum_{k=1}^{3}\left[\beta_k^{\frac{1}{\sigma-1}}A_k(t)\right]^{-a}\right\}^{-\frac{1}{a}} = \left\{\sum_{k=1}^{3}\tilde{\beta}_k e^{-ag_k(t-\lambda_k)}\right\}^{-\frac{1}{a}}, \quad where\ \tilde{\beta}_k \equiv \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^{a} > 0.$$

Longer adoption lags would shift down the time path of $U(t)$.

$$g_U(t) \equiv \frac{U'(t)}{U(t)} = \sum_{k=1}^{3} g_k s_k(t)$$

The aggregate growth rate is the weighted average of the sectoral growth rates

Relative Prices: $$\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \left[\left(\frac{\beta_j}{\beta_k}\right)^{-\alpha} \frac{\bar{A}_j(0)}{\bar{A}_k(0)}\right]^{-a} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{p_j(t)}{p_k(t)}\right)}{dt} = \frac{a(g_k - g_j)}{1-\sigma}$$

Relative Growth Effect: $p_j(t)/p_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $p_j(t)/p_k(t)$ at any point in time.
$Note$: For a fixed $\lambda_j > 0$, a higher g_j makes the relative price of j higher (though declining faster).

Relative Shares: $$\frac{s_j(t)}{s_k(t)} = \frac{\beta_j}{\beta_k}\left(\frac{p_j(t)}{p_k(t)}\right)^{1-\sigma} = \frac{\tilde{\beta}_j}{\tilde{\beta}_k} e^{a(\lambda_j g_j - \lambda_k g_k)} e^{a(g_k - g_j)t} \Longrightarrow \frac{d \ln\left(\frac{s_j(t)}{s_k(t)}\right)}{dt} = a(g_k - g_j)$$

Relative Growth Effect: $s_j(t)/s_k(t)$ is de(in)creasing over time if $g_j > (<) g_k$.
Shift from faster growing sectors to slower growing sectors over time.
Relative Level Effect: A higher $\lambda_j g_j - \lambda_k g_k$ raises $s_j(t)/s_k(t)$ at any point in time.
$Note$: For a fixed $\lambda_j > 0$, a higher g_j makes the relative share of j higher (though declining faster).

Structural Change with the Baumol (Relative Price) Effect: Let $g_1 > g_2 > g_3 > 0$

Decline of Agriculture: $s_1(t)$ is decreasing in t, because

$$\frac{1}{s_1(t)} - 1 = \frac{s_2(t)}{s_1(t)} + \frac{s_3(t)}{s_1(t)} = \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_1} e^{a(\lambda_2 g_2 - \lambda_1 g_1)}\right] e^{a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1} e^{a(\lambda_3 g_3 - \lambda_1 g_1)}\right] e^{a(g_1 - g_3)t}$$

Rise of Services: $s_3(t)$ is increasing in t, because

$$\frac{1}{s_3(t)} - 1 = \frac{s_1(t)}{s_3(t)} + \frac{s_2(t)}{s_3(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_3} e^{a(\lambda_1 g_1 - \lambda_3 g_3)}\right] e^{-a(g_1 - g_3)t} + \left[\frac{\tilde{\beta}_2}{\tilde{\beta}_3} e^{a(\lambda_2 g_2 - \lambda_3 g_3)}\right] e^{-a(g_2 - g_3)t}$$

Rise and Fall of Manufacturing: $s_2(t)$ is hump-shaped in t, because

$$\frac{1}{s_2(t)} - 1 = \frac{s_1(t)}{s_2(t)} + \frac{s_3(t)}{s_2(t)} = \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\lambda_1 g_1 - \lambda_2 g_2)}\right] e^{-a(g_1 - g_2)t} + \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\lambda_3 g_3 - \lambda_2 g_2)}\right] e^{a(g_2 - g_3)t}.$$

Hump-shaped due to the two opposing forces: $g_1 > g_2$ pushes labor out of A to M; $g_2 > g_3$ pulls labor out of M to S.

$$s_2'(t) \gtreqless 0 \Leftrightarrow (g_1 - g_2)\frac{s_1(t)}{s_2(t)} \gtreqless (g_2 - g_3)\frac{s_3(t)}{s_2(t)} \Leftrightarrow g_U(t) = \sum_{k=1}^{3} g_k s_k(t) \gtreqless g_2$$

Characterizing Manufacturing Peak: "^" indicates the peak.

$$s_2'(\hat{t}) = 0 \Leftrightarrow (g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t}) \Leftrightarrow g_U(\hat{t}) = g_2$$

Peak Time: From $(g_1 - g_2)s_1(\hat{t}) = (g_2 - g_3)s_3(\hat{t})$

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} + \hat{t}_0, \qquad \text{where } \hat{t}_0 \equiv \frac{1}{a(g_1 - g_3)} \ln\left[\left(\frac{g_1 - g_2}{g_2 - g_3}\right)\frac{\tilde{\beta}_1}{\tilde{\beta}_3}\right]$$

Two Normalizations: Without any loss of generality,

$$\hat{t}_0 = 0 \Leftrightarrow \frac{g_2 - g_3}{g_1 - g_2} = \frac{\tilde{\beta}_1}{\tilde{\beta}_3} \equiv \left[\left(\frac{\beta_1}{\beta_3}\right)^{\frac{1}{1-\sigma}} \frac{\bar{A}_3(0)}{\bar{A}_1(0)}\right]^a$$

The calendar time is reset so that its M-peak would be reached at $\hat{t} = 0$ in the absence of the adoption lags.

$$U(0) = 1 \text{ for } \lambda_1 = \lambda_2 = \lambda_3 = 0 \Leftrightarrow \sum\nolimits_{k=1}^{3} \tilde{\beta}_k = \sum\nolimits_{k=1}^{3} \left(\frac{\beta_k^{\frac{1}{1-\sigma}}}{\bar{A}_k(0)}\right)^a = 1.$$

We use the peak time per capita income in the absence of the adoption lags as the *numeraire.*

Note: Under these normalizations, the peak time share of sector-k in the absence of the adoption lags would be $\tilde{\beta}_k$.

Then,

Peak Time

$$\hat{t} = \frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3}.$$

Peak M-Share

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{\frac{a(g_1-g_2)(g_2-g_3)}{(g_1-g_3)}\left(\frac{\lambda_1 g_1 - \lambda_2 g_2}{g_1 - g_2} - \frac{\lambda_2 g_2 - \lambda_3 g_3}{g_2 - g_3}\right)}$$

Peak Time Per Capita Income

$$U(\hat{t}) = \left\{(1-\tilde{\beta}_2)e^{-a g_1 g_3\left(\frac{\lambda_1-\lambda_3}{g_1-g_3}\right)} + \tilde{\beta}_2 e^{-a g_2\left(\frac{\lambda_1 g_1 - \lambda_3 g_3}{g_1 - g_3} - \lambda_2\right)}\right\}^{-\frac{1}{a}}$$

So far, we have looked at the impacts of adoption lags in a single country in isolation, without specifying the sources of the adoption lags.

Next, we introduce cross-country heterogeneity, the technology gap, which generate cross-country variations in adoption lags, and study the cross-country implications.

Technology Gaps and Premature Deindustrialization

Consider the world with many countries with

$$(\lambda_1, \lambda_2, \lambda_3) = (\theta_1, \theta_2, \theta_3)\lambda$$

$\lambda \geq 0$: **Technology Gap,** Country-specific

$\theta_j > 0$: Sector-specific, capturing the inherent difficulty of technology adoption, common across countries

- **Countries differ only in one dimension**, λ, in their ability to adopt the frontier technologies.
- $\theta_j > 0$ determines how much the technology gap affects the adoption lag in that sector.

$$\frac{A_j(t)}{A_k(t)} = \frac{\bar{A}_j(0)}{\bar{A}_k(0)} e^{-(\theta_j g_j - \theta_k g_k)\lambda} e^{(g_j - g_k)t} \Rightarrow \frac{\partial}{\partial \lambda} \ln\left(\frac{A_j(t)}{A_k(t)}\right) = -(\theta_j g_j - \theta_k g_k)$$

Cross-country productivity difference is larger in sector-j than in sector-k if $\theta_j g_j > \theta_k g_k$.

Peak Time

$$\hat{t}(\lambda) = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda.$$

Peak M-Share

$$\frac{1}{\hat{s}_2(\lambda)} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{(g_2 - g_3)\left(\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}\right)a\lambda}$$

Peak Time Per Capita Income

$$\hat{U}(\lambda) = \left\{(1 - \tilde{\beta}_2)e^{-g_1 g_3\left(\frac{\theta_1 - \theta_3}{g_1 - g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3} - \theta_2\right)a\lambda}\right\}^{-\frac{1}{a}}$$

Figure 1: Conditions for Premature Deindustrialization (PD) only with the Baumol (Relative Price) Effect

$$\hat{t}'(\lambda) > 0 \text{ for all } \lambda > 0 \Leftrightarrow \theta_1 g_1 > \theta_3 g_3.$$

With $\theta_1 g_1 > \theta_3 g_3$, the price of A is relatively higher than the price of S in a high-λ country, which delays the peak.

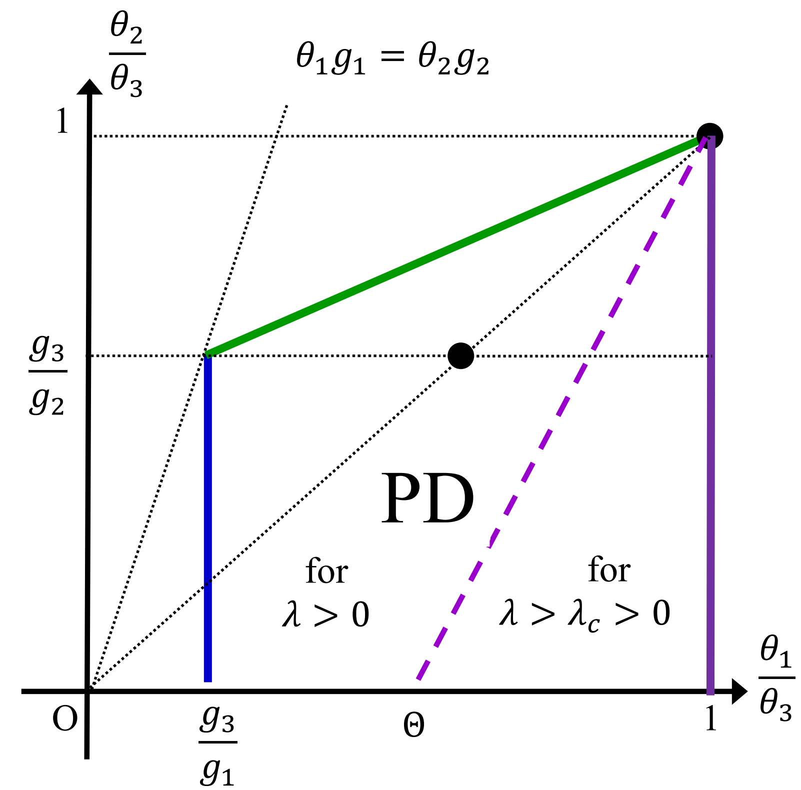
$$\hat{s_2}'(\lambda) < 0 \text{ for all } \lambda > 0 \Leftrightarrow \frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} > \frac{\theta_2 g_2 - \theta_3 g_3}{g_2 - g_3}$$

With a low θ_2, which has no effect on $\hat{t}$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\hat{U}'(\lambda) < 0;\ \hat{U}(\lambda) < \hat{U}(0) \text{ for } \lambda > \lambda_c \geq 0 \Leftrightarrow \theta_1 < \theta_3 \Leftrightarrow \hat{t}(\lambda) < \theta_1 \lambda < \theta_3 \lambda$$

With $\theta_1 < \theta_3$, the time delay in the peak in a high-λ country is not long enough to make up for the lagging productivity, that is deindustrialization is "premature."



These conditions jointly imply $\theta_1 g_1 > \theta_2 g_2, \theta_3 g_3$ (productivity differences the largest in A) and $\theta_1, \theta_2 < \theta_3$ (adoption lag the longest in S).

Some Examples

Example 1: No Premature Deindustrialization (PD)

Uniform Adoption Lags, as in Krugman (1985)

$$\theta_1 = \theta_2 = \theta_3 = 1 \iff \lambda_1 = \lambda_2 = \lambda_3 = \lambda > 0$$

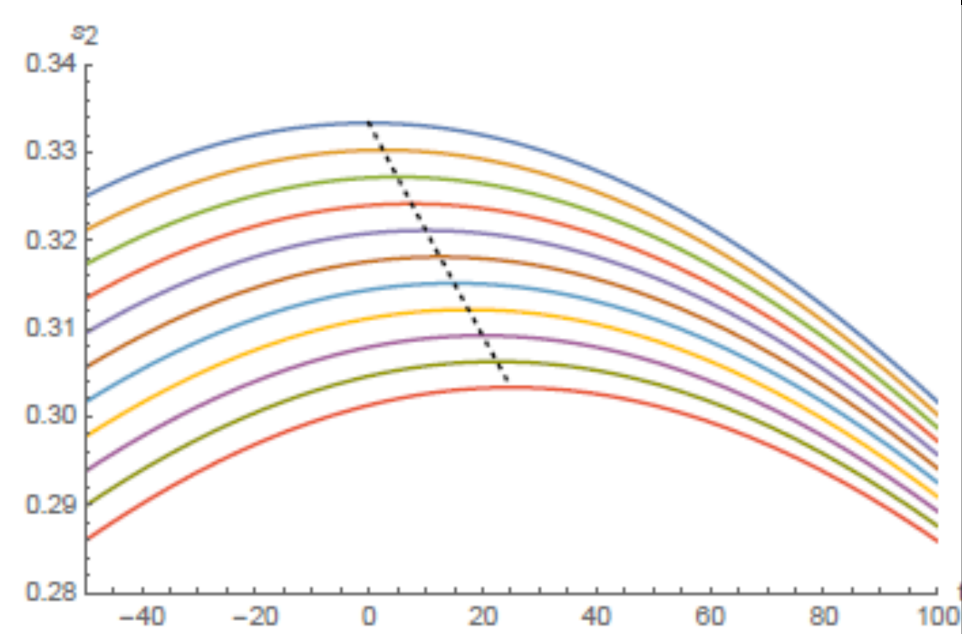
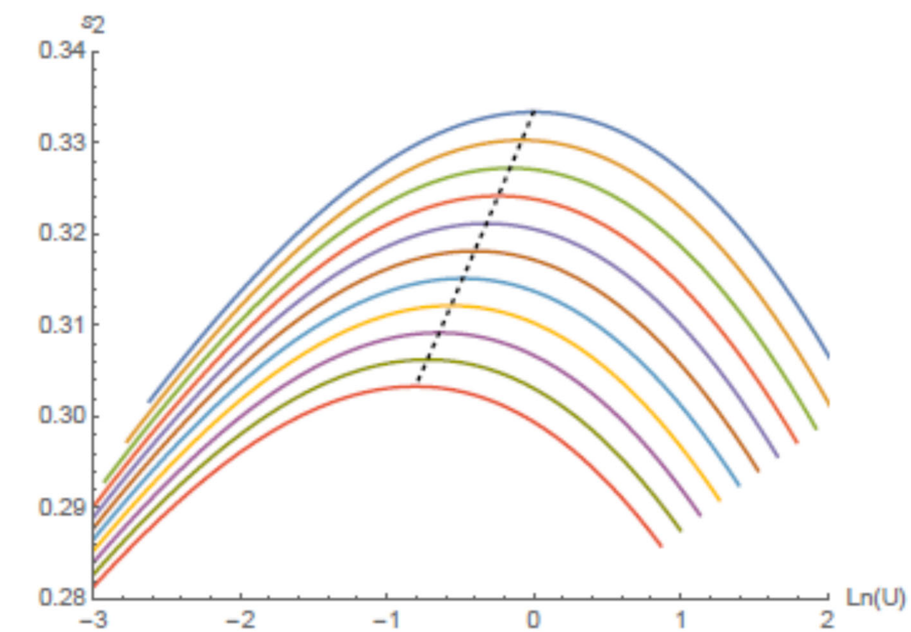
$$\Longrightarrow \hat{t}(\lambda) = \lambda; \quad \widehat{s_2}(\lambda) = \tilde{\beta}_2; \quad \widehat{U}(\lambda) = 1$$

- The country's technology gap causes a delay in the peak time, $\hat{t}$, by $\lambda > 0$.
- The peak M-share & the peak time per capita income unaffected.

Each country follows exactly the same development path of early industrializers *with a delay*. No PD!!

Thus, the technology gap must have differential impacts on the adoption lags across sectors.

Example 2a-2c: Numerical Illustrations. In all three examples, $\theta_1 = \theta_2 < \theta_3 = 1$ and we use $g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$; $\alpha = 1/3$, and $\sigma = 0.6$ (hence $a = 6/13$).
$\tilde{\beta}_j = 1/3$ for $j = 1,2,3 \Rightarrow \widehat{s_2}(0) = \tilde{\beta}_2 = 1/3$; $\widehat{U}(0) = 1$; $\hat{t}(0) = 0$.

Example 2a	$(t, s_2(t))$	$(\ln U(t), s_2(t))$
$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.5 = \frac{g_3}{g_2}$$ $$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 = \theta_3 g_3$$ Cross-country productivity differences are the same in M & in S in this case.		
λ=0 λ=10 λ=20 λ=30 λ=40 λ=50 λ=60 λ=70 λ=80 λ=90 λ=100		

<table>
<tr><th></th><th>$\left(t, s_2(t)\right)$</th><th>$\left(\ln U(t), s_2(t)\right)$</th></tr>
<tr><td>

Example 2b

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.35 < \frac{g_3}{g_2}$$

$$\Longrightarrow \theta_1 g_1 > \theta_3 g_3 > \theta_2 g_2$$

Cross-country productivity differences the smallest in M.

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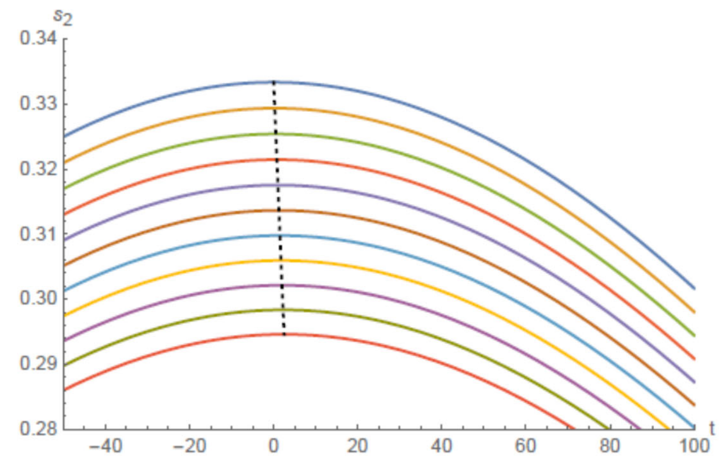

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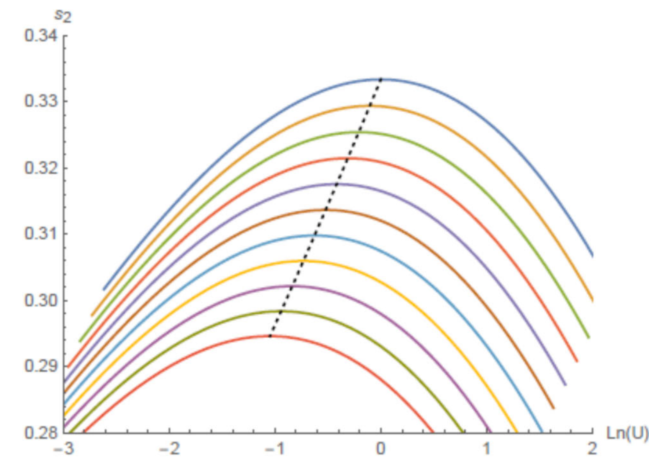

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Example 2c

$$\frac{\theta_1}{\theta_3} = \frac{\theta_2}{\theta_3} = 0.75 > \frac{g_3}{g_2}$$

$$\Longrightarrow \theta_1 g_1 > \theta_2 g_2 > \theta_3 g_3$$

Cross-country productivity differences the smallest in S.

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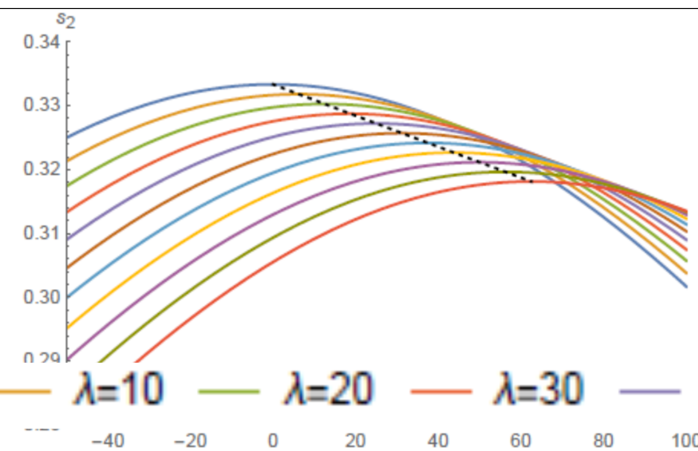

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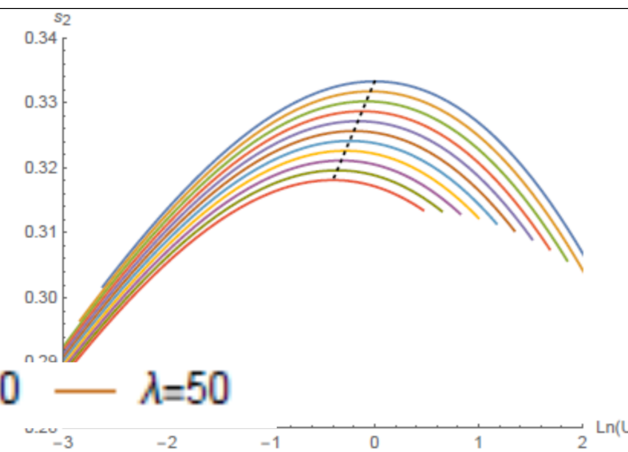

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λ=0 λ=10 λ=20 λ=30 λ=40 λ=50

λ=60 λ=70 λ=80 λ=90 λ=100

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Rodrik (2016) divided countries into pre-1990 peaked vs. post-1990 peaked.
From his Fig.5, $\hat{t}(\lambda) = 25$ years.
For the employment shares (Fig.6), $\widehat{s_2}(0) = 22\% > \widehat{s_2}(\lambda) = 19\%$; $\ln \widehat{U}(0) = 0 > \ln \widehat{U}(\lambda) = -0.95$
For the value-added shares (Fig.7), $\widehat{s_2}(0) = 28\% > \widehat{s_2}(\lambda) = 24\%$; $\ln \widehat{U}(0) = 0 > \ln \widehat{U}(\lambda) = -0.83$.

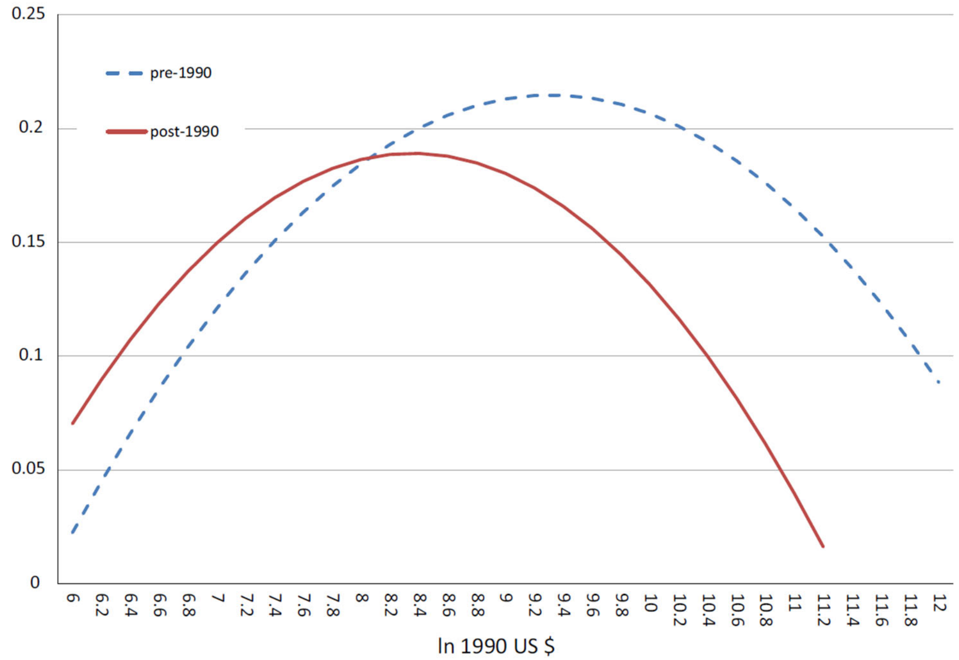


Fig. 6 Simulated manufacturing employment shares

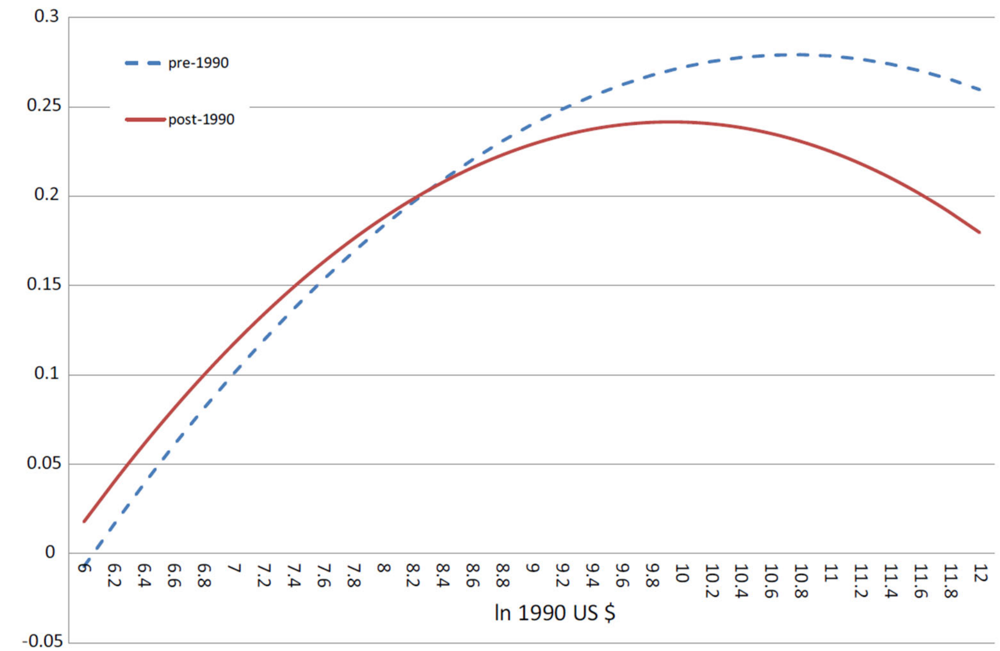
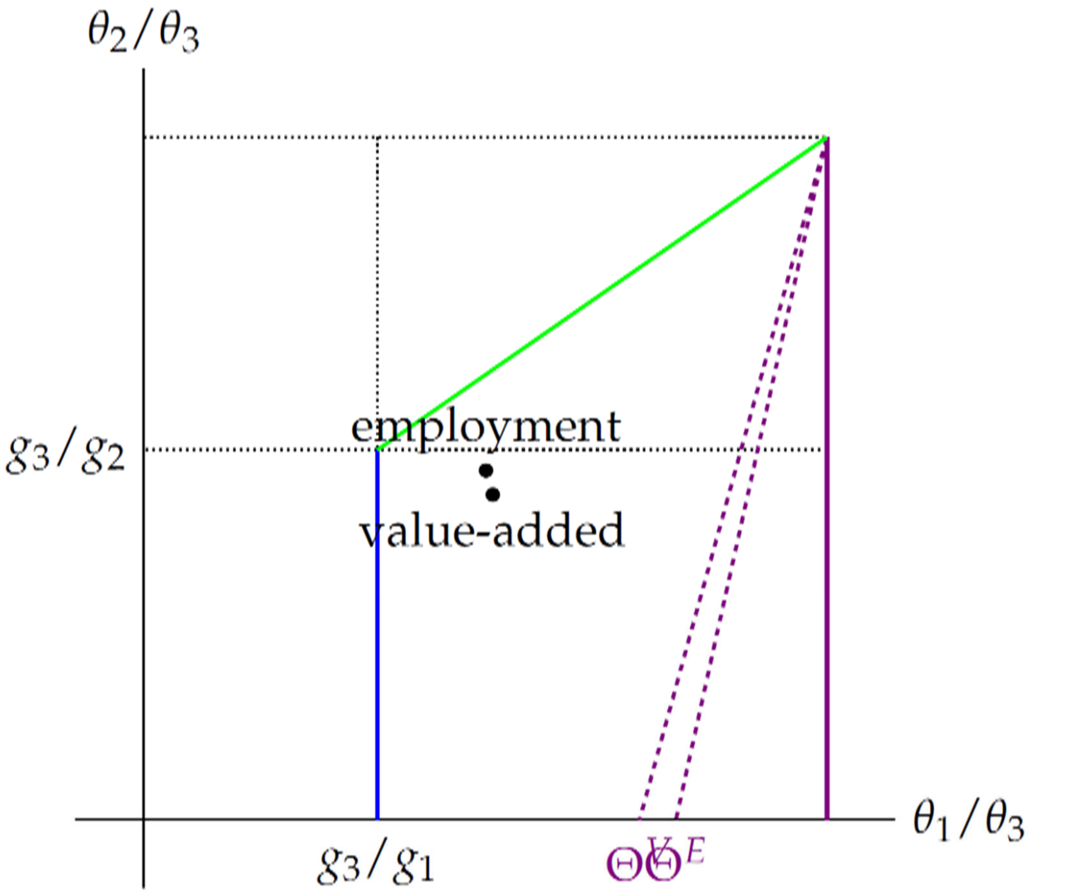
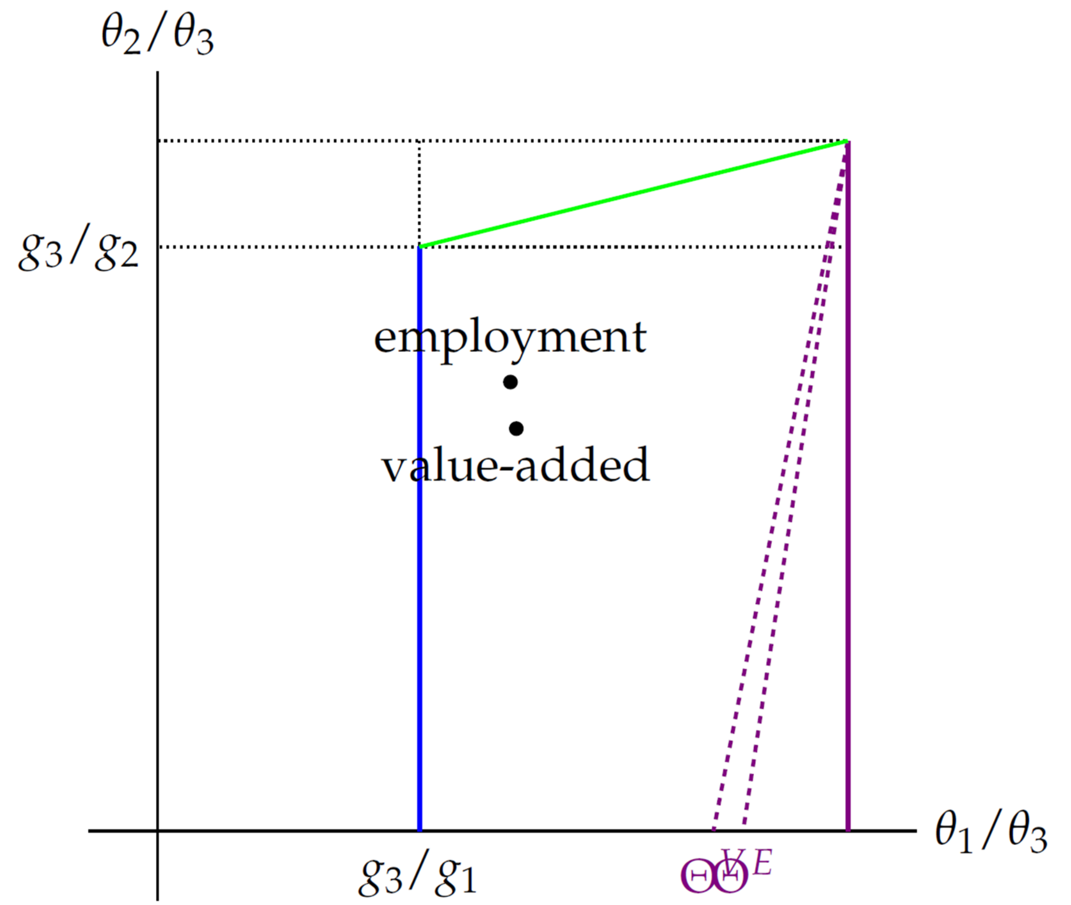


Fig. 7 Simulated manufacturing output shares (MVA/GDP at constant prices)

Duarte-Restuccia (2010) $g_1 = 3.8\% > g_2 = 2.4\% > g_3 = 1.3\%$	**Comin, Lashkari, and Mestieri (2021)** $g_1 = 2.9\% > g_2 = 1.3\% > g_3 = 1.1\%$
$\theta_1/\theta_3 = 0.51;\ \theta_2/\theta_3 = 0.51;\ \Theta = 0.78$ for employment shares $\theta_1/\theta_3 = 0.51;\ \theta_2/\theta_3 = 0.48;\ \Theta = 0.73$ for value-added shares	$\theta_1/\theta_3 = 0.51;\ \theta_2/\theta_3 = 0.65;\ \Theta = 0.85$ for employment shares $\theta_1/\theta_3 = 0.52;\ \theta_2/\theta_3 = 0.58;\ \Theta = 0.81$ for value-added shares
	

Introducing the Engel Effect

The Engel Law through Isoelastic Nonhomothetic CES; Comin-Lashkari-Mestieri (2021), Matsuyama (2019)

$$\left[\sum_{j=1}^{3}(\beta_j)^{\frac{1}{\sigma}}\left(\frac{c_j}{U^{\varepsilon_j}}\right)^{1-\frac{1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \equiv 1$$

Normalize $\varepsilon_1 + \varepsilon_2 + \varepsilon_3 = 3$; with $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we go back to the standard homothetic CES.
With $\sigma < 1$, $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 \Rightarrow$ **the income elasticity the lowest in A and the highest in S.**

By maximizing U subject to $\sum_{j=1}^{3} p_j c_j \leq E$,

Expenditure Shares $$m_j \equiv \frac{p_j c_j}{E} = \frac{\beta_j\left(U^{\varepsilon_j}p_j\right)^{1-\sigma}}{\sum_{k=1}^{3}\beta_k(U^{\varepsilon_k}p_k)^{1-\sigma}} = \beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma} \Longrightarrow \frac{m_j}{m_k} = \frac{\beta_j}{\beta_k}\left(\frac{p_j}{p_k}U^{\varepsilon_j-\varepsilon_k}\right)^{1-\sigma}$$

Indirect Utility Function: $$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j}p_j}{E}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1$$

Cost-of-Living Index: $$\left[\sum_{j=1}^{3}\beta_j\left(\frac{U^{\varepsilon_j-1}p_j}{P}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}} \equiv 1 \Leftrightarrow U \equiv \frac{E}{P}$$

Income Elasticity: $$\eta_j \equiv \frac{\partial \ln c_j}{\partial \ln(U)} = 1 + \frac{\partial \ln m_j}{\partial \ln(E/P)} = 1 + (1-\sigma)\left\{\varepsilon_j - \sum_{k=1}^{3} m_k\varepsilon_k\right\}$$

Structural Change with the Engel (Income) Effect: Let $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$.
Then, even with constant relative prices,

Decline of Agriculture: $s_1(t) = m_1(t)$ is decreasing in $U(t)$, because

$$\frac{1}{s_1(t)} - 1 = \frac{m_2(t)}{m_1(t)} + \frac{m_3(t)}{m_1(t)} = \frac{\beta_2}{\beta_1}\left(\frac{p_2}{p_1}U(t)^{\varepsilon_2-\varepsilon_1}\right)^{1-\sigma} + \frac{\beta_3}{\beta_1}\left(\frac{p_3}{p_1}U(t)^{\varepsilon_3-\varepsilon_1}\right)^{1-\sigma}$$

Rise of Services: $s_3(t) = m_3(t)$ is increasing in $U(t)$, because

$$\frac{1}{s_3(t)} - 1 = \frac{m_1(t)}{m_3(t)} + \frac{m_2(t)}{m_3(t)} = \frac{\beta_1}{\beta_3}\left(\frac{p_1}{p_3}U(t)^{\varepsilon_1-\varepsilon_3}\right)^{1-\sigma} + \frac{\beta_2}{\beta_3}\left(\frac{p_2}{p_3}U(t)^{\varepsilon_2-\varepsilon_3}\right)^{1-\sigma}$$

Rise and Fall of Manufacturing: $s_2(t) = m_2(t)$ is hump-shaped in $U(t)$, because

$$\frac{1}{s_2(t)} - 1 = \frac{m_1(t)}{m_2(t)} + \frac{m_3(t)}{m_2(t)} = \frac{\beta_1}{\beta_2}\left(\frac{p_1}{p_2}U(t)^{\varepsilon_1-\varepsilon_2}\right)^{1-\sigma} + \frac{\beta_3}{\beta_2}\left(\frac{p_3}{p_2}U(t)^{\varepsilon_3-\varepsilon_2}\right)^{1-\sigma}.$$

Hump-shaped due to the two opposing forces: $\varepsilon_1 < \varepsilon_2$ pushes labor out of A to M; $\varepsilon_2 < \varepsilon_3$ pulls labor out of M to S.

$$s_2'(t) = m_2'(t) \gtreqless 0 \Leftrightarrow (\varepsilon_2 - \varepsilon_1)\frac{m_1(t)}{m_2(t)} \gtreqless (\varepsilon_3 - \varepsilon_2)\frac{m_3(t)}{m_2(t)} \Leftrightarrow \eta_2 \gtreqless 1$$

with constant relative prices.

The production side is the same as before. By following the same step, we obtain

Equilibrium Shares

$$s_j = \frac{\left[\beta_j^{\frac{1}{\sigma-1}} A_j\right]^{-a}}{[U^{\varepsilon_j}]^{-a}}, \qquad \text{where } \sum_{k=1}^{3} \frac{\left[\beta_k^{\frac{1}{\sigma-1}} A_k\right]^{-a}}{[U^{\varepsilon_k}]^{-a}} \equiv 1$$

With $A_j(t) = \bar{A}_j(t - \lambda_j) = \bar{A}_j(0) e^{g_j(t-\theta_j \lambda)}$,

$s_2(t)$:
$$\frac{1}{s_2(t)} = U(t)^{a(\varepsilon_1-\varepsilon_2)} \left[\frac{\tilde{\beta}_1}{\tilde{\beta}_2} e^{a(\theta_1 g_1 - \theta_2 g_2)\lambda}\right] e^{-a(g_1-g_2)t} + 1 + U(t)^{a(\varepsilon_3-\varepsilon_2)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_2} e^{a(\theta_3 g_3 - \theta_2 g_2)\lambda}\right] e^{a(g_2-g_3)t}$$

$U(t)$:
$$U(t)^{a\varepsilon_1} \tilde{\beta}_1 e^{-a g_1 (t-\theta_1 \lambda)} + U(t)^{a\varepsilon_2} \tilde{\beta}_2 e^{-a g_2 (t-\theta_2 \lambda)} + U(t)^{a\varepsilon_3} \tilde{\beta}_3 e^{-a g_3 (t-\theta_3 \lambda)} \equiv 1$$

$s_2'(t) = 0$:
$$(g_1 - g_2) = (g_2 - g_3) U^{a(\varepsilon_3-\varepsilon_2)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3 - \theta_1 g_1)\lambda} e^{a(g_1-g_3)t}$$
$$+ \frac{\left\{(\varepsilon_1 - \varepsilon_2) + (\varepsilon_3 - \varepsilon_2) U^{a(\varepsilon_3-\varepsilon_1)} \left[\frac{\tilde{\beta}_3}{\tilde{\beta}_1}\right] e^{a(\theta_3 g_3 - \theta_1 g_1)\lambda} e^{a(g_1-g_3)t}\right\} \left\{g_1 U^{a(\varepsilon_1-\varepsilon_2)} \tilde{\beta}_1 e^{-a g_1 (t-\theta_1\lambda)} + g_2 \tilde{\beta}_2 e^{-a g_2 (t-\theta_2\lambda)} + g_3 U^{a(\varepsilon_3-\varepsilon_2)} \tilde{\beta}_3 e^{-a g_3 (t-\theta_3\lambda)}\right\}}{\varepsilon_1 U^{a(\varepsilon_1-\varepsilon_2)} \tilde{\beta}_1 e^{-a g_1 (t-\theta_1\lambda)} + \varepsilon_2 \tilde{\beta}_2 e^{-a g_2 (t-\theta_2\lambda)} + \varepsilon_3 U^{a(\varepsilon_3-\varepsilon_2)} \tilde{\beta}_3 e^{-a g_3 (t-\theta_3\lambda)}}.$$

$\hat{t}$ and $\hat{U}$ solve the equation for $U(t)$ and the equation for $s_2'(t) = 0$, simultaneously.
Then, $\hat{s}_2$ can be obtained by plugging $\hat{t}$ and $\hat{U}$ into the equation for $s_2(t)$

(Analytically Solvable) "Unbiased" Case

$$0 < \mu \equiv \frac{\varepsilon_2 - \varepsilon_1}{g_1 - g_2} = \frac{\varepsilon_3 - \varepsilon_2}{g_2 - g_3} < \frac{1}{g_1 - \bar{g}}, \qquad \text{where} \qquad \bar{g} \equiv \frac{g_1 + g_2 + g_3}{3}$$

Peak Time

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda - \ln\left\{(1-\tilde{\beta}_2)e^{-g_1 g_3\left(\frac{\theta_1-\theta_3}{g_1-g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1-\theta_3 g_3}{g_1-g_3}-\theta_2\right)a\lambda}\right\}^{-\frac{1}{a}\left(\frac{\mu}{1+\mu\bar{g}}\right)}$$

Peak M-Share

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{1}{\tilde{\beta}_2} - 1\right)e^{(g_2-g_3)\left(\frac{\theta_1 g_1-\theta_3 g_3}{g_1-g_3} - \frac{\theta_2 g_2-\theta_3 g_3}{g_2-g_3}\right)a\lambda}$$

Peak Time Per Capita Income

$$U(\hat{t}) = \left\{(1-\tilde{\beta}_2)e^{-g_1 g_3\left(\frac{\theta_1-\theta_3}{g_1-g_3}\right)a\lambda} + \tilde{\beta}_2 e^{-g_2\left(\frac{\theta_1 g_1-\theta_3 g_3}{g_1-g_3}-\theta_2\right)a\lambda}\right\}^{-\frac{1}{a}\left(\frac{1}{1+\mu\bar{g}}\right)}$$

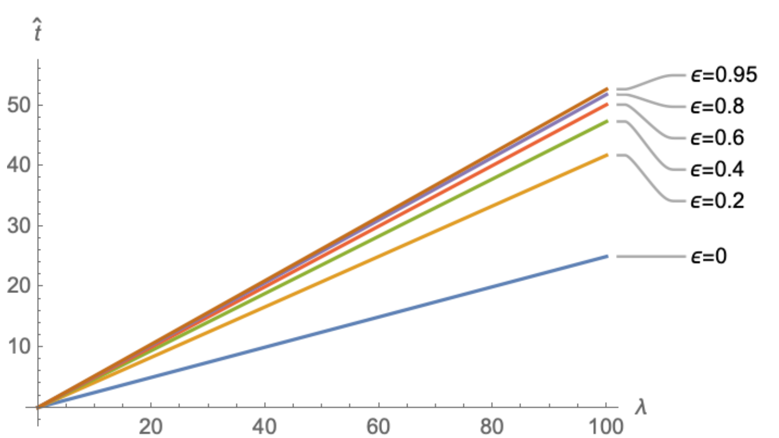
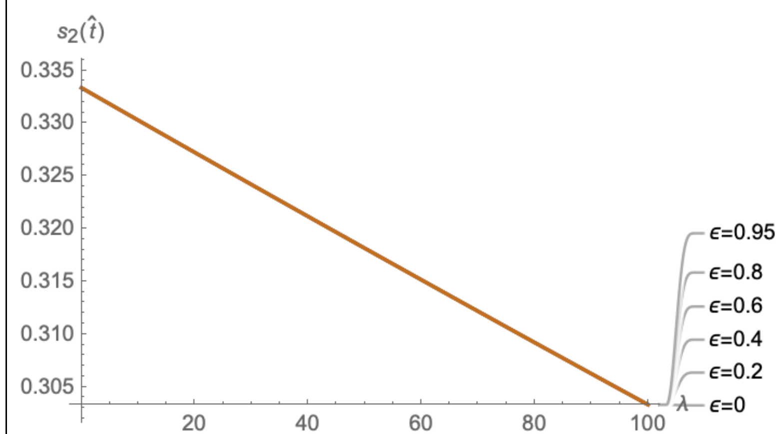
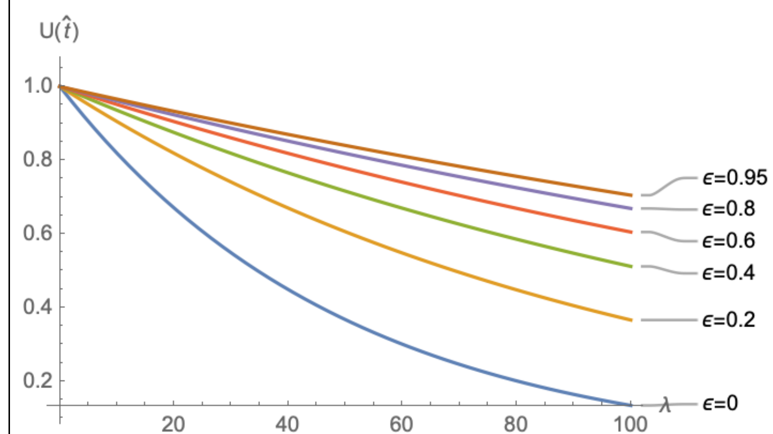
$\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0$; $\frac{\partial U(\hat{t})}{\partial \lambda} < 0$ under the same condition; $\frac{\partial \hat{t}}{\partial \lambda} > 0$ under a weaker condition. With g_1, g_2, g_3 fixed, a higher μ has

- **No effect** on $\hat{t}, s_2(\hat{t}), U(\hat{t})$ for the country with $\lambda = 0$.
- A further delay in $\hat{t}$ for every country with $\lambda > 0$.
- **No effect on** $s_2(\hat{t})$ for every country with $\lambda > 0$.
- A smaller decline in $U(\hat{t})$ for each country with $\lambda > 0$.

(Analytically Solvable) "Unbiased" Case: A Numerical Illustration

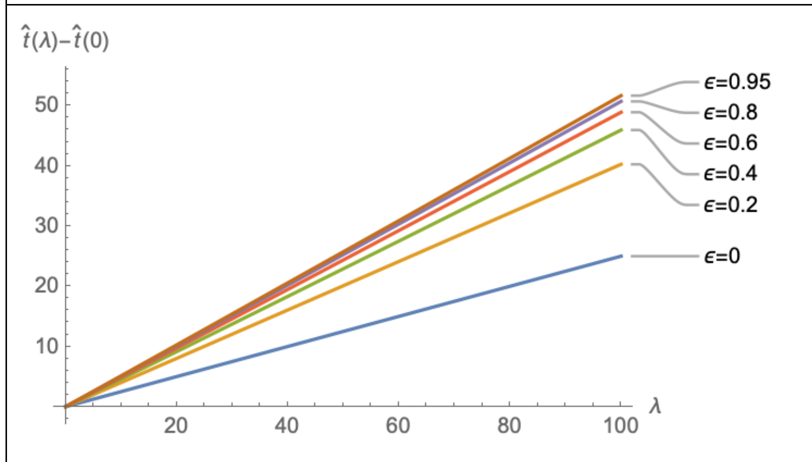
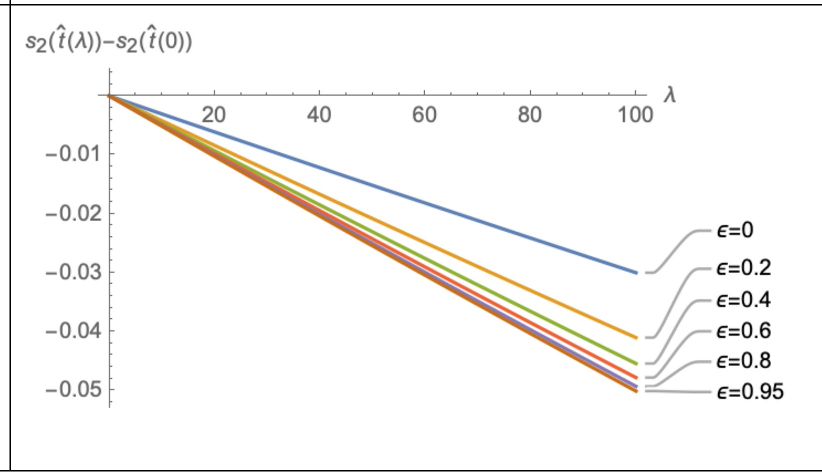
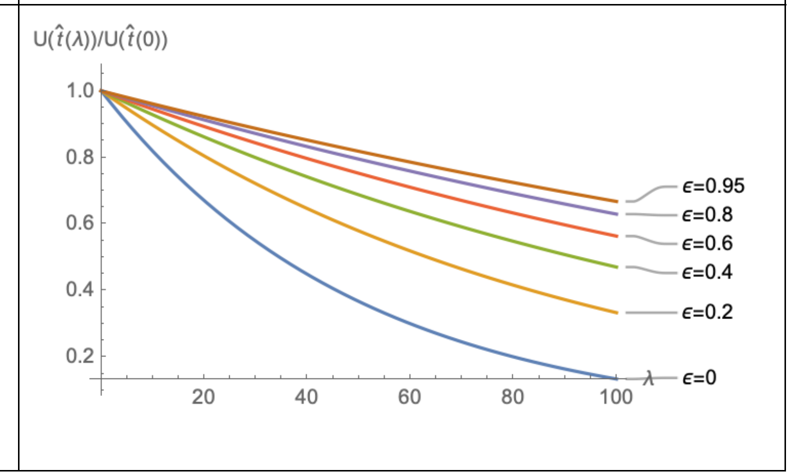
$g_1 = 3.6\% > g_2 = 2.4\% > g_3 = 1.2\%$, $\theta = 0.5$, $a = 6/13$; $\tilde{\beta}_j = 1/3$ for $j = 1,2,3$.

In this case, $g_1 - g_2 = g_2 - g_3 = \bar{g} = 1.2\% > 0 \Longrightarrow \varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$ for $0 < \epsilon = (1.2\%)\mu < 1$

Peak Time	Peak M-Share	Peak Time Per Capita Income
		

(Empirically More Plausible) Biased Case:

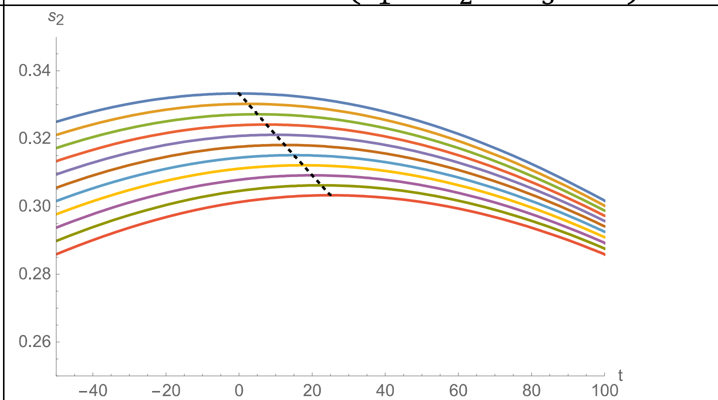
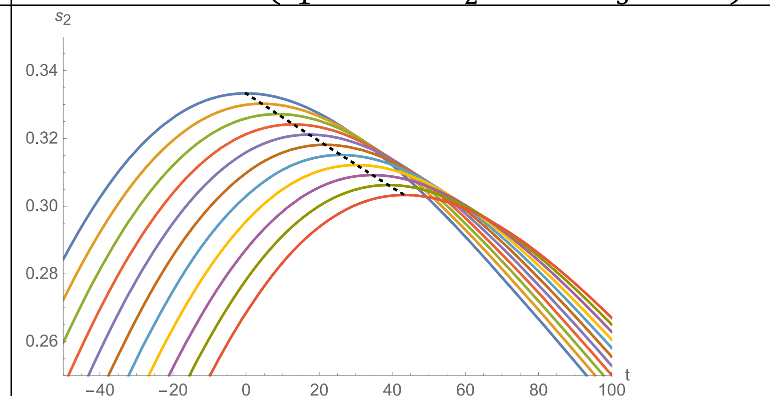
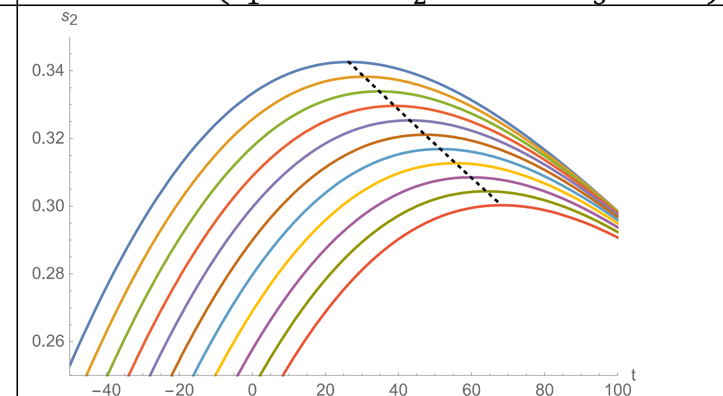
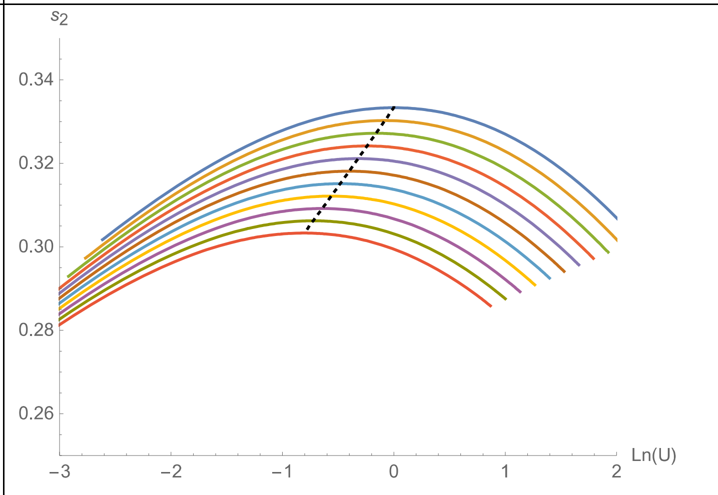
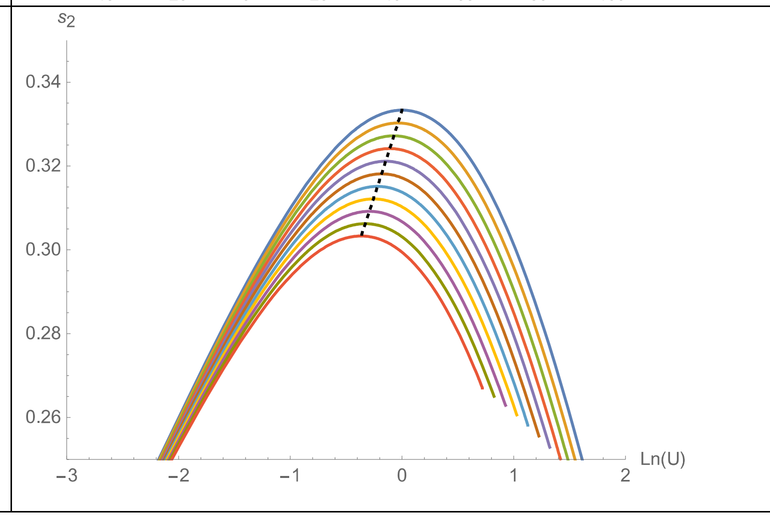
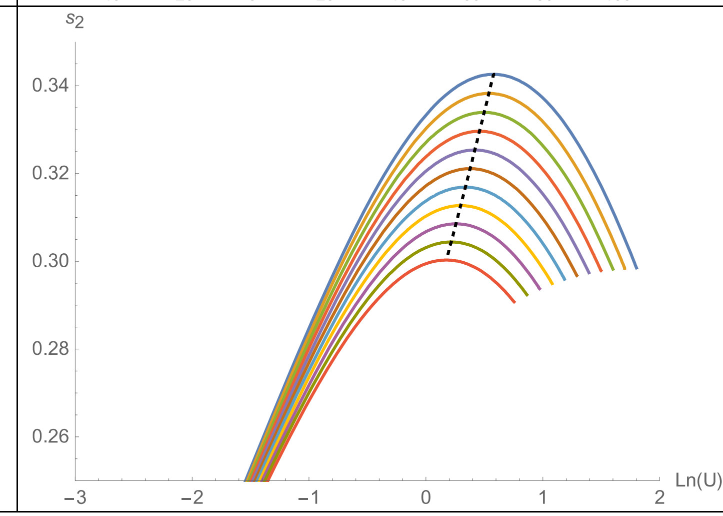
$\varepsilon_1 = 1 - \epsilon < \ \varepsilon_2 = 1 + \frac{\epsilon}{3} < \varepsilon_3 = 1 + \frac{2\epsilon}{3}$ for $0 < \epsilon < 1 \Longrightarrow \frac{g_1 - g_2}{g_2 - g_3} = 1 < \frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2} = 4$, as in CLM (2021).

Peak Time	**Peak M-Share**	**Peak Time Per Capita Income**
		

In this case, the frontier country's peak values are affected by ϵ. Relative to the frontier country, a higher ϵ causes a high-λ country to have

- A further delay in $\hat{t}$
- A *larger* decline in $s_2(\hat{t})$.
- A smaller decline in $U(\hat{t})$.

Stronger nonhomotheticity changes the shape of the time paths significantly.
It does not change the implications on PD, i.e., how technology gaps affect $\hat{t}, s_2(\hat{t})$, and $U(\hat{t})$.

	$\epsilon = 0$	$\epsilon = 0.6$	
	Homothetic case ($\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$)	Unbiased case($\varepsilon_1 = .4 < \varepsilon_2 = 1 < \varepsilon_3 = 1.6$)	Biased case($\varepsilon_1 = .4 < \varepsilon_2 = 1.2 < \varepsilon_3 = 1.4$)
$(t, s_2(t))$			
$(\ln U(t), s_2(t))$			

Premature Deindustrialization (PD) through the Engel (Income) Effect Only

What happens if we had *solely* the Engel effect with $0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 = 3 - \varepsilon_1 - \varepsilon_2$, without the Baumol effect, $g_1 = g_2 = g_3 = \bar{g} > 0$?

Peak Time

$$\hat{t} = \frac{1}{a\bar{g}} \ln\left\{(1 - \tilde{\beta}_2) e^{\frac{(\varepsilon_3\theta_1 - \varepsilon_1\theta_3)}{(\varepsilon_3 - \varepsilon_1)} a\bar{g}\lambda} + \tilde{\beta}_2 e^{\left(\theta_2 + \frac{(\theta_1 - \theta_3)}{(\varepsilon_3 - \varepsilon_1)}\varepsilon_2\right) a\bar{g}\lambda}\right\}$$

Peak M-Share

$$\frac{1}{s_2(\hat{t})} - 1 = \left(\frac{1}{\tilde{\beta}_2} - 1\right) e^{(\varepsilon_3 - \varepsilon_2)\left(\frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} - \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2}\right) a\bar{g}\lambda}$$

Peak Time Per Capita Income

$$\ln U(\hat{t}) = \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} \bar{g}\lambda$$

with the two normalizations

$$\left(\frac{\varepsilon_2 - \varepsilon_1}{\varepsilon_3 - \varepsilon_2}\right) \frac{\tilde{\beta}_1}{\tilde{\beta}_3} = 1; \;\; \tilde{\beta}_1 + \tilde{\beta}_2 + \tilde{\beta}_3 = 1$$

which ensures $U(\hat{t}) = 1$ and $\hat{t} = 0$ for $\lambda = 0$.

Conditions for Premature Deindustrialization (PD) only with the Engel Effect

$$\frac{\partial U(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow 0 < \frac{\theta_1}{\theta_3} < 1$$

With a low θ_1 and a high θ_3, the price of the income elastic S is high relative to the income inelastic A in a high-λ country, which make it necessary to reallocate labor to S at earlier stage of development.

$$\frac{\partial s_2(\hat{t})}{\partial \lambda} < 0 \text{ for all } \lambda > 0 \Leftrightarrow \frac{\theta_1 - \theta_3}{\varepsilon_3 - \varepsilon_1} > \frac{\theta_2 - \theta_3}{\varepsilon_3 - \varepsilon_2}$$

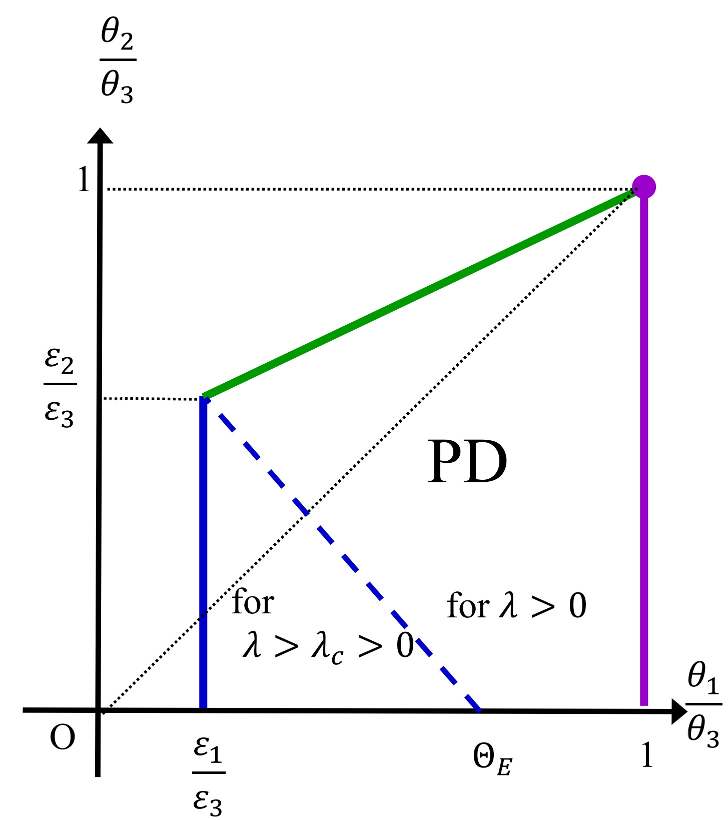
With a low θ_2, which has no effect on $U(\hat{t})$, the price of M is low relative to both A & S in a high-λ country, which keeps the M-share low.

Under the above condition,

$$\frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ for a sufficiently large } \lambda \Leftrightarrow \frac{\theta_1}{\theta_3} > \frac{\varepsilon_1}{\varepsilon_3}$$

$$\frac{\partial \hat{t}}{\partial \lambda} > 0 \text{ for all } \lambda > 0 \Leftrightarrow \left(\Theta_E - \frac{\varepsilon_1}{\varepsilon_3}\right)\left[1 - \left(\frac{\varepsilon_3}{\varepsilon_2}\right)\frac{\theta_2}{\theta_3}\right] < \frac{\theta_1}{\theta_3} - \frac{\varepsilon_1}{\varepsilon_3} < 1 - \frac{\varepsilon_1}{\varepsilon_3}$$

where $\varepsilon_1/\varepsilon_3 < \Theta_E < 1$.



With $g_1 = g_2 = g_3 = \bar{g}$, PD occurs only if $\theta_1\bar{g}, \theta_2\bar{g} < \theta_3\bar{g}$, that is, when cross-country productivity difference is *the largest in S.*

Introducing Catching Up

Narrowing a Technology Gap

We assumed that λ is time-invariant. This implies

The sectoral productivity growth rate is constant over time & identical across countries.
[In contrast, the aggregate growth rate, $g_U(t) \equiv U'(t)/U(t) = \sum_{k=1}^{3} g_k s_k(t)$, declines over time, $g_U'(t) = g_1 s_1'(t) + g_2 s_2'(t) + g_3 s_3'(t) = (g_1 - g_2)s_1'(t) + (g_3 - g_2)s_3'(t) < 0$, the so-called Baumol's cost disease.]

*What if technological laggards can **narrow a technology gap**, and hence achieve a higher productivity growth in each sector?*

Countries differ only in the *initial* value of lambda, $\boldsymbol{\lambda_0}$, converging exponentially over time at **the same rate,**

$$A_j(t) = \bar{A}_j(0)e^{g_j(t-\theta_j\lambda_t)}, \quad \text{where } \lambda_t = \lambda_0 e^{-g_\lambda t}, \quad g_\lambda > 0.$$

$$\Longrightarrow \frac{1}{s_2(t)} = \left(\frac{\tilde{\beta}_1}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_1 g_1 - \theta_2 g_2)\lambda_t - (g_1 - g_2)t]} + 1 + \left(\frac{\tilde{\beta}_3}{\tilde{\beta}_2}\right) e^{\alpha[(\theta_3 g_3 - \theta_2 g_2)\lambda_t + (g_2 - g_3)t]}$$

Again, by setting the calendar time such that $\hat{t}_0 = 0$ for the frontier country with $\lambda_0 = 0$,

Peak Time

$$\hat{t} = \frac{\theta_1 g_1 - \theta_3 g_3}{g_1 - g_3}\lambda_{\hat{t}} + D(g_\lambda \lambda_{\hat{t}})$$

Peak Share

$$\frac{1}{s_2(\hat{t})} = 1 + \left(\frac{\tilde{\beta}_1 + \tilde{\beta}_3}{\tilde{\beta}_2}\right)\left[\frac{(g_2 - g_3)e^{a(g_2-g_1)D(g_\lambda\lambda_{\hat{t}})} + (g_1 - g_2)e^{a(g_2-g_3)D(g_\lambda\lambda_{\hat{t}})}}{g_1 - g_3}\right]\left[e^{\frac{a(g_1-g_2)(g_2-g_3)}{(g_1-g_3)}}\right]^{\left(\frac{\theta_1 g_1 - \theta_2 g_2}{g_1 - g_2} + \frac{\theta_3 g_3 - \theta_2 g_2}{g_2 - g_3}\right)\lambda_{\hat{t}}}$$

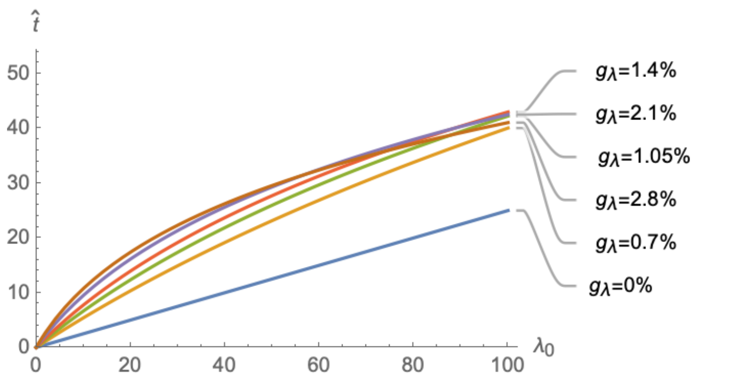
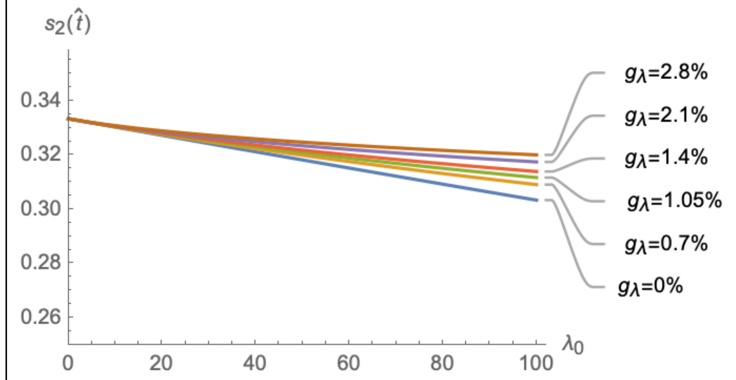
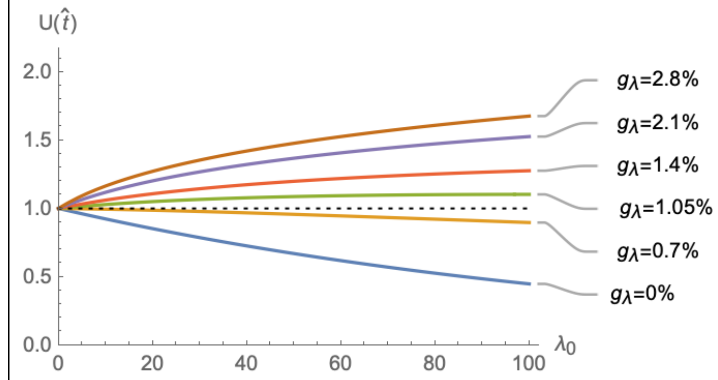
Peak Time Per Capita Income

$$U(\hat{t}) = \left\{\left(\tilde{\beta}_1 e^{-ag_1 D(g_\lambda\lambda_{\hat{t}})} + \tilde{\beta}_3 e^{-ag_3 D(g_\lambda\lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1-\theta_3)g_1 g_3}{g_1-g_3}\lambda_{\hat{t}}} + \left(\tilde{\beta}_2 e^{-ag_2 D(g_\lambda\lambda_{\hat{t}})}\right)e^{-a\frac{(\theta_1-\theta_2)g_1 g_2 + (\theta_2-\theta_3)g_2 g_3}{g_1-g_3}\lambda_{\hat{t}}}\right\}^{-\frac{1}{a}}$$

where

$$D(g_\lambda\lambda_{\hat{t}}) = \frac{1}{a(g_1 - g_3)}\ln\left[\left(\frac{g_1 - g_2 + (\theta_1 g_1 - \theta_2 g_2)g_\lambda\lambda_{\hat{t}}}{g_2 - g_3 - (\theta_3 g_3 - \theta_2 g_2)g_\lambda\lambda_{\hat{t}}}\right)\left(\frac{g_2 - g_3}{g_1 - g_2}\right)\right].$$

For $g_\lambda = 0, D(g_\lambda\lambda_{\hat{t}}) = D(0) = 0$, and all the parts in red disappear, and we go back to the baseline model.

Peak Time	Peak M-Share	Peak time Per Capita Income
		

Technological laggards

- peak later in time,
- have lower peak M-shares
- have lower peak time per capita income, unless g_λ is too large: Comin-Mestieri (2018)

Concluding Remarks

A Parsimonious model of Rodrik's (2016) PD based on

- **Differential productivity growth rates across complementary sectors**, as in Baumol (67), Ngai-Pissarides (07).
- **Countries heterogeneous only in their technology gaps**, as in Krugman (1985).
- Sectors differ in the extent to which technology gap affects their adoption lags, unlike in Krugman (1985)

We find that PD occurs for

- cross-country productivity difference larger in A than in S.
- technology adoption takes not too long in M.
- Technology adoption takes longer in S than in A.

which implies that cross-country productivity difference the largest in A; that technology adoption the longest in S.

The baseline model assumes **homothetic CES (**to focus on the Baumol effect) and **no catching up** (to isolate the level effect from the growth effect).

In two extensions, we showed that the results are ***robust*** against introducing

- **The Engel effect** with income-elastic S & income-inelastic A, using nonhomothetic CES: CLM(21), Matsuyama(19)

The Engel effect changes the shape of the time paths, but not the implications on technology gaps on PD

The Engel effect *alone* could not generate PD w/o counterfactual implications on cross-country productivity differences

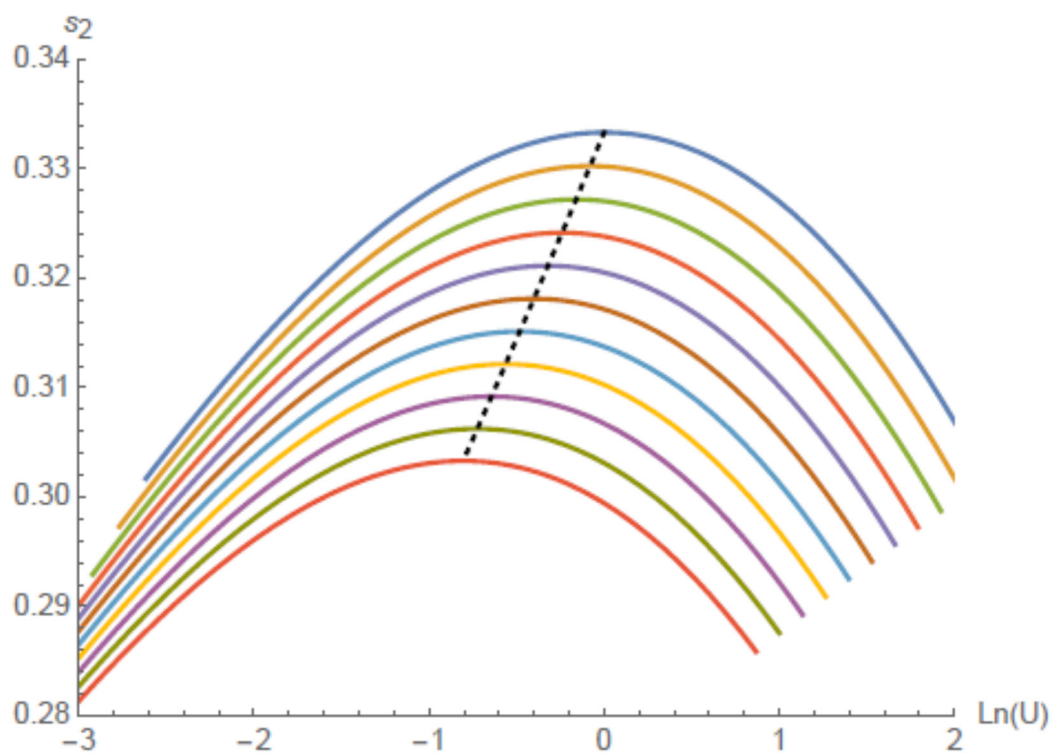
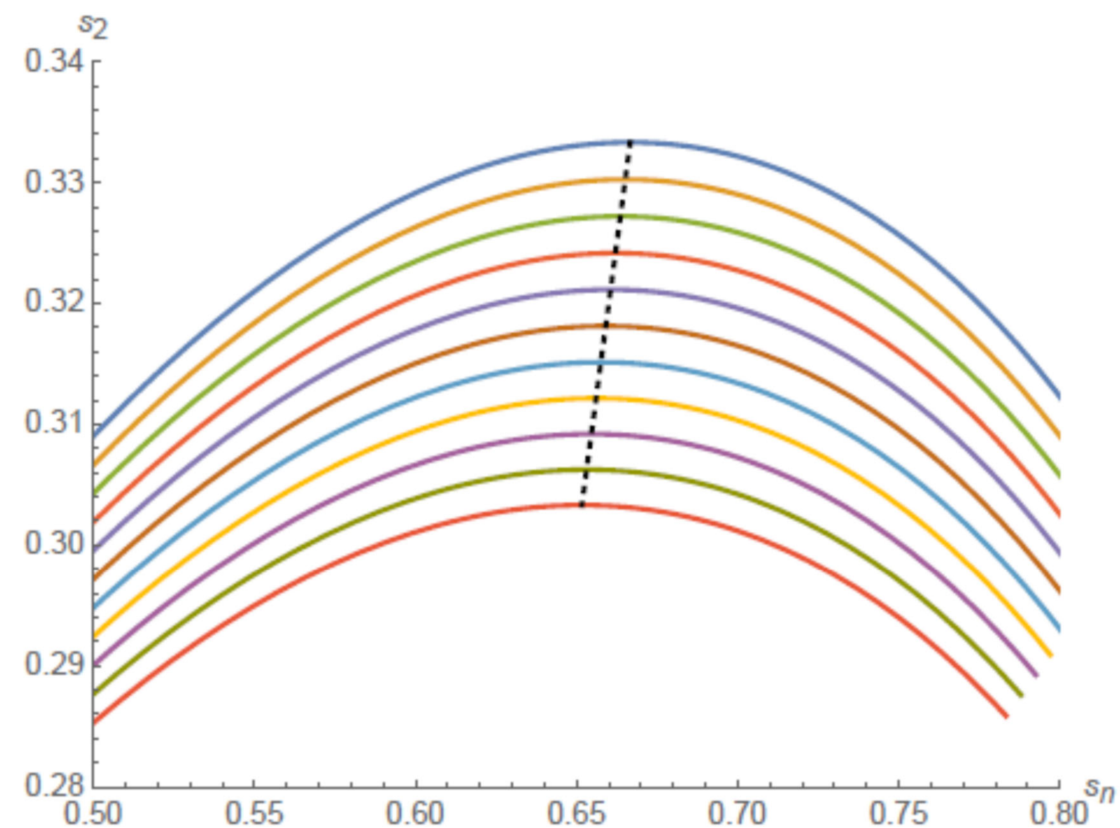
- **Narrowing a technology gap** to allow technological laggards to catch up

unless the catching-up speed is too large.

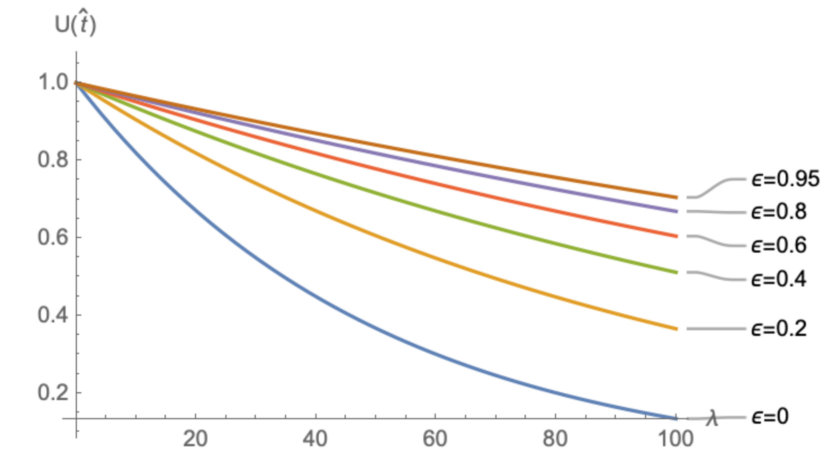
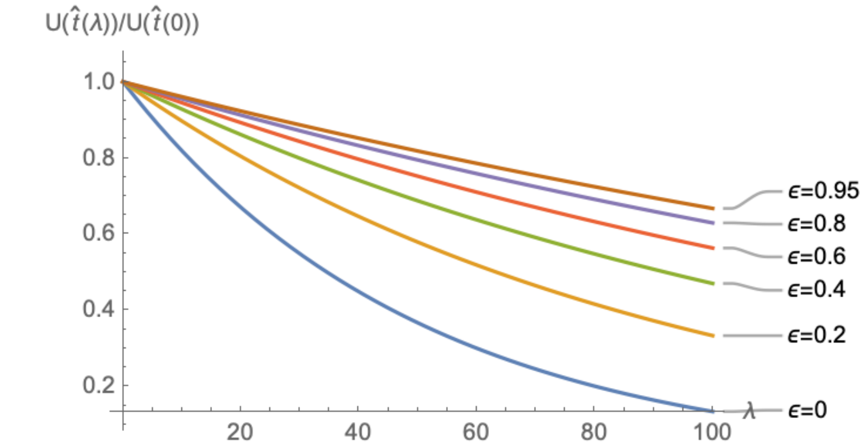
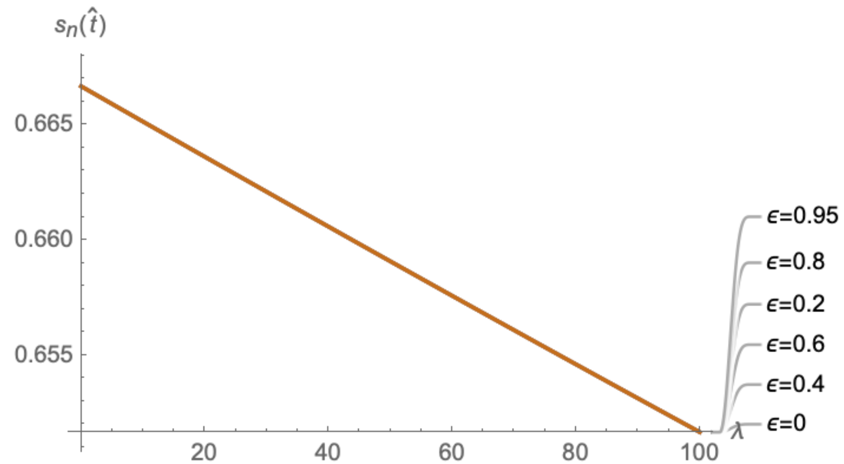
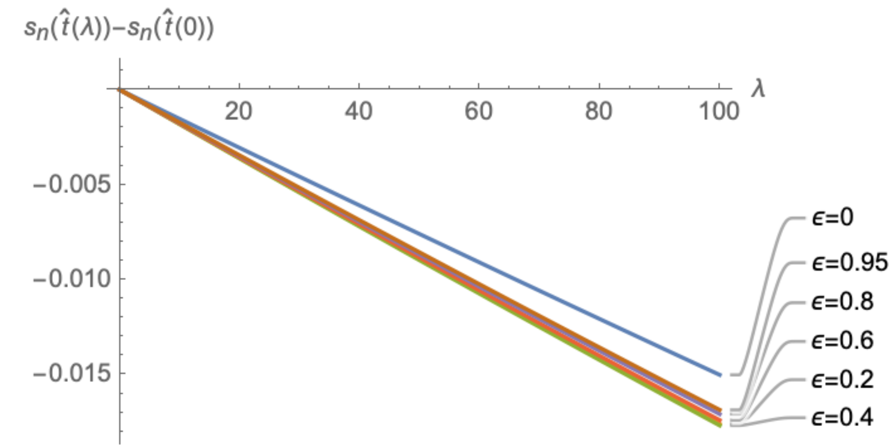
Appendix

Appendix: Non-agricultural share as another measure of development, $1 - s_1(\hat{t}) = s_2(\hat{t}) + s_3(\hat{t}) \equiv s_n(\hat{t})$

Baseline Homothetic Case:

$\left(\ln U(t), s_2(t)\right)$	$\left(s_n(t), s_2(t)\right)$
	

Nonhomothetic Cases:

<table>
<tr><th></th><th>Unbiased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 < \varepsilon_3 = 1 + \epsilon$</th><th>Biased: $\varepsilon_1 = 1 - \epsilon < \varepsilon_2 = 1 + \epsilon/3 < \varepsilon_3 = 1 + 2\epsilon/3$</th></tr>
<tr><td>$\ln U(t)$</td><td>
</td><td>
</td></tr>
<tr><td>$s_n(t)$</td><td>
</td><td>
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In the biased case, the frontier country's peak values are affected by ϵ.